## Guided Practice 2.11

**Example 1**

The starting balance of Anna's account is $1,250. She takes $30 out of her account each month. How much money is in her account after 1, 2, and 3 months? Find an explicit function to represent the balance in her account at any month.

1. Use the description of the account balance to find the balance after each month.

   Anna's account has $1,250. After 1 month, she takes out $30, so her account balance decreases by $30: $1250 – $30 = $1220.

   The new starting balance of Anna's account is $1,220. After 2 months, she takes out another $30. Subtract this $30 from the new balance of her account: $1220 – $30 = $1190.

   The new starting balance of Anna's account is $1,190. After 3 months, she takes out another $30. Subtract this $30 from the new balance of her account: $1190 – $30 = $1160.

2. Determine the independent and dependent quantities.

   The month number is the independent quantity, since the account balance depends on the month. The account balance is the dependent quantity.

3. Determine if there is a common difference or common ratio that describes the change in the dependent quantity.

   Organize your results in a table. Enter the independent quantity in the first column, and the dependent quantity in the second column. The balance at zero months is the starting balance of the account, before any money has been taken out. Because the independent quantity is changing by one month at a time, analyzing the differences between the dependent quantities will determine if there is a common difference between the dependent quantities.

| Month | Account balance in dollars ($) | Difference |
|---|---|---|
| 0 | 1250 | |
| 1 | 1220 | 1250 –1220 = –30 |
| 2 | 1190 | 1220 –1190 = –30 |
| 3 | 1160 | 1190 –1160 = –30 |

   The account balance has a common difference; it decreases by $30 for every 1 month. The relationship between the month and the account balance can be represented using a linear function.

4. Use the common difference to write an explicit function.

   The slope-intercept form of a linear function is $f(x) = mx + b$, where $m$ is the slope and $b$ is the $y$-intercept. The common difference between the dependent terms in the pattern is the slope of the relationship between the independent and dependent quantities. Replace $m$ with the slope, and replace $x$ and $f(x)$ with an ordered pair from the table, such as (1, 1220). Solve for $b$.

   $1220 = (-30) \bullet (1) + b$

   $1250 = b$

   $f(x) = -30x + 1250$

   The explicit function for this scenario is $f(x) = -30x + 1250$.

5. Evaluate the function to verify that it is correct.

   Organize your results in a table. Use the explicit function to find each term. The terms that are calculated should match the terms in the original list.

| **Month, $x$** | **Account balance, $f(x)$, in dollars ($)** |
|---|---|
| 0 | $(-30) \bullet (0) + 1250 = 1250$ |
| 1 | $(-30) \bullet (1) + 1250 = 1220$ |
| 2 | $(-30) \bullet (2) + 1250 = 1190$ |
| 3 | $(-30) \bullet (3) + 1250 = 1160$ |

   The pairs of dependent and independent quantities match the ones in the original pattern, so the explicit function is correct.

   The balance in Anna's account can be represented using the function $f(x) = -30x + 1250$.

**Example 2**

A video arcade charges an entrance fee, then charges a fee per game played. The entrance fee is $5, and each game costs an additional $1. Find the total cost for playing 0, 1, 2, or 3 games. Describe the total cost of playing $x$ games with an explicit function.

1. Use the description of the costs to find the total costs.

   If no games are played, then only the entrance fee is paid. The total cost for playing 0 games is $5.

   If 1 game is played, then the entrance fee is paid, plus the cost of one game. If each game is $1, the cost of one game is $1. The total cost is $5 + $1 = $6.

   If 2 games are played, then the entrance fee is paid, plus the cost of two games. If each game is $1, the cost of two games is $1 • 2 = $2. The total cost is $5 + $2 = $7.

   If 3 games are played, then the entrance fee is paid, plus the cost of three games. If each game is $1, the cost of three games is $1 • 3 = $3. The total cost is $5 + $3 = $8.

2. Identify the independent and dependent quantities.

   The total cost is dependent on the number of games played, so the number of games is the independent quantity and the total cost is the dependent quantity.

3. Determine if there is a common difference or a common ratio between the dependent terms.

   There appears to be a common difference between the dependent terms. Use a table to find the difference between the dependent quantities. Subtract the current term from the previous term.

| Games | Cost in dollars ($) | Difference |
|---|---|---|
| 0 | 5 | |
| 1 | 6 | $6 - 5 = 1$ |
| 2 | 7 | $7 - 6 = 1$ |
| 3 | 8 | $8 - 7 = 1$ |

   The common difference between the dependent terms is $1.

4. Use the common difference to write an explicit function.

   The slope-intercept form of a linear function is $f(x) = mx + b$, where $m$ is the slope and $b$ is the $y$-intercept. The common difference between the dependent terms in the pattern is the slope of the relationship between the independent and dependent quantities. Replace $m$ with the slope, and replace $x$ and $f(x)$ with a coordinate pair from the table, such as (1, 6). Solve for $b$.

   $6 = (1) \bullet (1) + b$

   $5 = b$

   The explicit function is $f(x) = 1x + 5$.

5. Evaluate the function to verify that it is correct.

   Organize your results in a table. Use the explicit function to find each term. The terms that are calculated should match the terms in the original list.

| Games | Cost in dollars ($) |
|---|---|
| 0 | $1 \bullet (0) + 5 = 5$ |
| 1 | $1 \bullet (1) + 5 = 6$ |
| 2 | $1 \bullet (2) + 5 = 7$ |
| 3 | $1 \bullet (3) + 5 = 8$ |

   The pairs of independent and dependent quantities match the ones in the original pattern, so the explicit function is correct.

   The total cost of any number of games, $x$, can be represented using the function $f(x) = x + 5$.