Guided Practice 2.12

Example 1

Find the common difference, write the explicit formula, and find the tenth term for the following arithmetic sequence.

3, 9, 15, 21, ...

1. Find the common difference by subtracting two successive terms.

   $9 - 3 = 6$

2. Confirm that the difference is the same between each remaining pair of consecutive terms.

   $15 - 9 = 6$ and $21 - 15 = 6$

3. Identify the first term, a_1.

   $a_1 = 3$

4. Write the explicit formula.

   $a_n = a_1 + (n - 1)d$ Explicit formula for an arithmetic sequence

   $a_n = (3) + (n - 1)(6)$ Substitute 3 for a_1 and 6 for d.

5. Simplify the explicit formula.

   $a_n = 3 + 6n - 6$ Distribute 6 over $(n - 1)$.

   $a_n = 6n - 3$ Simplify.

6. To find the tenth term, substitute 10 for n in the explicit formula.

   $a_n = 6n - 3$ Explicit formula from the previous step

   $a_{(10)} = 6(10) - 3$ Substitute 10 for n.

   $a_{10} = 60 - 3$ Multiply.

   $a_{10} = 57$ Subtract.

   The tenth term in the sequence is 57.

Instruction

Example 2

Write a linear function that corresponds to the following arithmetic sequence.

8, 1, –6, –13, ...

1. Find the common difference by subtracting two successive terms.

   $1 - 8 = -7$

2. Confirm that the difference is the same between each remaining pair of consecutive terms.

   $-6 - 1 = -7$ and $-13 - (-6) = -7$

3. Identify the first term, a_1.

   $a_1 = 8$

4. Write the explicit formula.

   $a_n = a_1 + (n - 1)d$ Explicit formula for an arithmetic sequence

   $a_n = (8) + (n - 1)(-7)$ Substitute 8 for a_1 and –7 for d.

5. Simplify the explicit formula.

   $a_n = 8 - 7n + 7$ Distribute –7 over $(n - 1)$.

   $a_n = -7n + 15$ Simplify.

6. Write the explicit formula in function notation.

   $f(x) = -7x + 15$

   Note that the domain of an arithmetic sequence is positive consecutive integers.

Instruction

Example 3

An arithmetic sequence is defined recursively by the formula $a_n = a_{n-1} + 5$, with $a_1 = 29$. Find the first 5 terms of the sequence, write an explicit formula to represent the sequence, and find the 15th term.

1. Use the recursive formula, beginning with a_1, to calculate the next 4 terms.

   We are given that the first term, a_1, is 29. Substitute 2, 3, 4, and 5, respectively, for n in the recursive formula $a_n = a_{n-1} + 5$ to find the next 4 terms.

   $a_1 = 29$

   $a_2 = 29 + 5 = 34$

   $a_3 = 34 + 5 = 39$

   $a_4 = 39 + 5 = 44$

   $a_5 = 44 + 5 = 49$

   The first 5 terms of the sequence are 29, 34, 39, 44, and 49.

2. Write the explicit formula for this sequence.

   The first term is $a_1 = 29$ and the common difference is $d = 5$, so the explicit formula is $a_n = 29 + (n - 1)5$.

3. Simplify the explicit formula.

   $a_n = 29 + 5n - 5$ Distribute 5 over $(n - 1)$.

   $a_n = 5n + 24$ Simplify.

4. Find the requested term in the sequence.

   Substitute 15 for n in the explicit formula to find the 15th term.

   $a_n = 5n + 24$ Explicit formula from the previous step

   $a_{(15)} = 5(15) + 24$ Substitute 15 for n.

   $a_{15} = 75 + 24$ Multiply.

   $a_{15} = 99$ Add.

   The 15th term in the sequence is 99.