

Lesson 4.11: Comparing Linear to Exponential Functions

Instruction

Guided Practice 4.11

Example 1

Which function increases faster, $f(x) = 4x - 5$ or $g(x) = 4^x - 5$? Justify your answer with a graph.

1. Make a general observation.

$f(x) = 4x - 5$ is a linear function of the form $f(x) = mx + b$.

The variable x is multiplied by the coefficient 4.

$g(x) = 4^x - 5$ is an exponential function of the form $g(x) = ab^x + k$.

The variable x is the exponent.

2. Create a table of values.

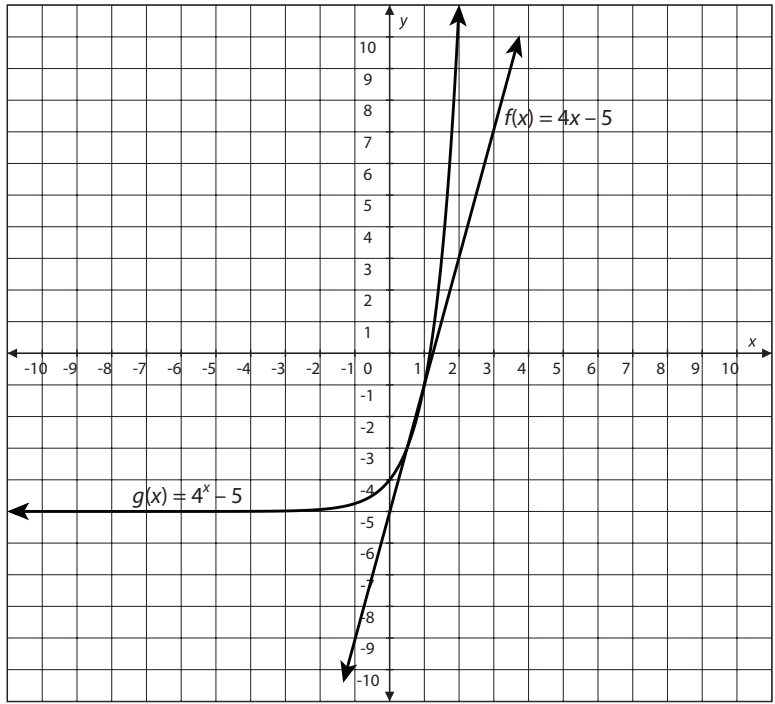
Substitute values for x into each function.

$f(x) = 4x - 5$		$g(x) = 4^x - 5$	
x	$f(x)$	x	$g(x)$
-2	-13	-2	-4.9375
-1	-9	-1	-4.75
0	-5	0	-4
1	-1	1	-1
2	3	2	11

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- Graph both functions on the same coordinate plane.
Use the tables of values created in order to plot both functions.



- Compare the rate of change of each function.
The graph of $f(x) = 4x - 5$ appears to be steeper than the graph of $g(x) = 4^x - 5$ until the point $(1, -1)$. At this point, the graphs intersect and $f(x) = g(x)$. Once x is greater than 1, the graph of $g(x) = 4^x - 5$ becomes steeper. From there, $g(x) = 4^x - 5$ increases faster than $f(x) = 4x - 5$. ✓

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Example 2

At approximately what point does the value of $f(x)$ exceed the value of $g(x)$ if $f(x) = 2(4)^{\frac{x}{20}}$ and $g(x) = 0.5x$? Justify your answer with a graph.

1. Make a general observation.

$f(x) = 2(4)^{\frac{x}{20}}$ is an exponential function of the form $g(x) = ab^x$.

The variable x is part of the exponent.

$g(x) = 0.5x$ is a linear function of the form $f(x) = mx + b$.

The variable x is multiplied by the coefficient 0.5.

2. Create a table of values.

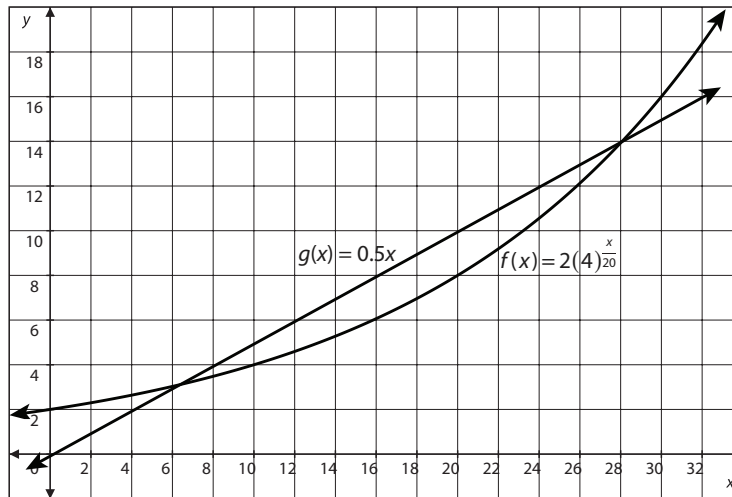
Substitute values for x into each function.

$f(x) = 2(4)^{\frac{x}{20}}$		$g(x) = 0.5x$	
x	$f(x)$	x	$g(x)$
0	2	0	0
2	2.30	2	1
4	2.64	4	2
6	3.03	6	3

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- Graph both functions on the same coordinate plane.
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- Identify the approximate point where $f(x)$ is greater than $g(x)$.
It can be seen from the graph that both functions are equal where x is approximately equal to 28. When x is greater than 28, $f(x)$ is greater than $g(x)$.



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Example 3

Lena has been offered a job with two salary options. The first option is modeled by the function $f(x) = 500x + 31,000$, where $f(x)$ is her salary in dollars after x years. The second option is represented by the function $g(x) = 29,000(1.04)^x$, where $g(x)$ is her salary in dollars after x years. If Lena is hoping to keep this position for at least 5 years, which salary option should she choose? Support your answer with a graph.

1. Make a general observation.

$f(x) = 500x + 31,000$ is a linear function of the form $f(x) = mx + b$.

The variable x is multiplied by the coefficient 500 and added to the constant 31,000.

$g(x) = 29,000(1.04)^x$ is an exponential function of the form $g(x) = ab^x$.

The variable x is the exponent.

Use the two equations to create a table of values.

Substitute the same values for x into each function.

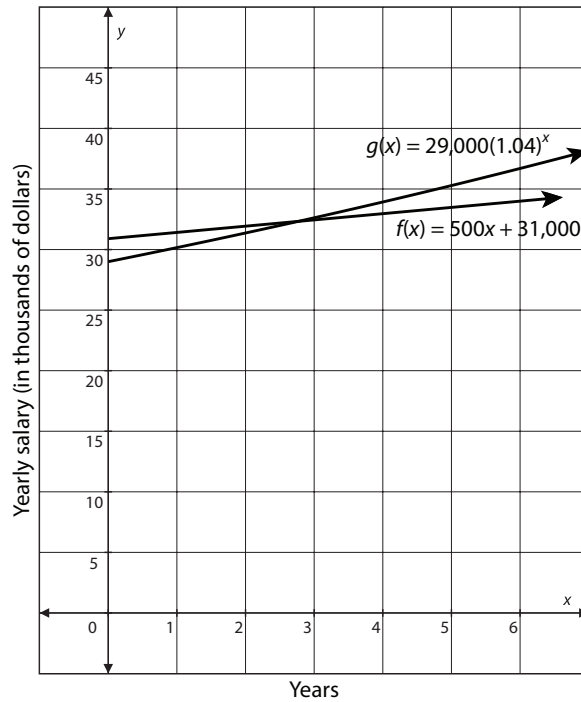
$f(x) = 500x + 31,000$		$g(x) = 29,000(1.04)^x$	
x	$f(x)$	x	$g(x)$
0	31,000	0	29,000
2	32,000	2	31,366.40
4	33,000	4	33,925.90
6	34,000	6	36,694.25



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- Graph both functions on the same coordinate plane.
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- Identify the approximate point where $g(x)$ is greater than $f(x)$.

It can be seen from the graph that after 3 years, $g(x)$ is greater than $f(x)$. If Lena is hoping to keep this position for at least 5 years, it is in her best interest to choose the salary option modeled by $g(x) = 29,000(1.04)^x$.

