Guided Practice 4.5

Example 1

A 20 kg mixture is evaporating at a rate of 3% of its mass each hour.

- Determine whether the scenario described represents growth or decay.
- Determine the percent of increase or decrease.
- Determine the growth or decay factor.
- Write the exponential growth or decay model.
- Sketch the graph of the relationship.
- Describe what the domain, the range, and the point (0,20) represent.

1. Determine whether the scenario described represents growth or decay.

   Because the mixture is evaporating and is losing part of its mass each hour, the scenario represents decay.

2. Determine the percent of increase or decrease.

   The percent of decrease is 3%. The rate written as a decimal is 0.03.

3. Determine the growth or decay factor.

$b = 1 - r$	Formula for decay factor
$b = 1 - 0.03$	Substitute 0.03 for r, the rate of decay.
$b = 0.97$	Subtract to find b, the decay factor.

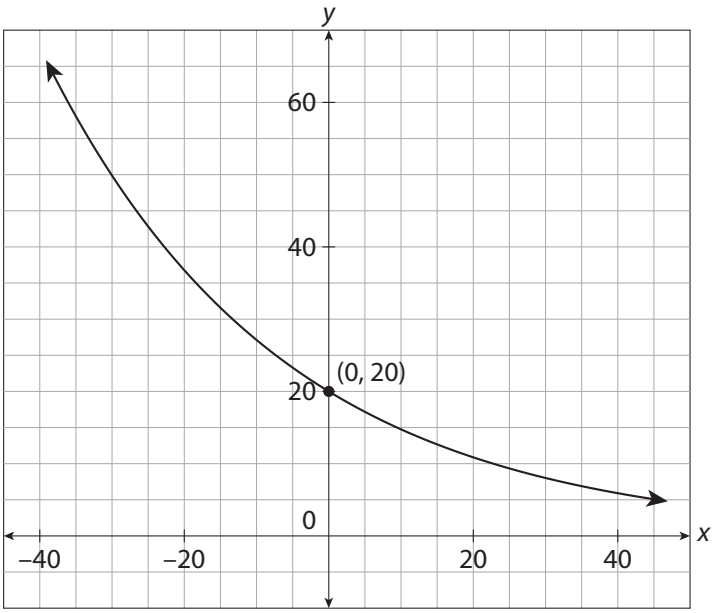
Instruction

4. Write the exponential growth or decay model.

$y = a(b)^x$	Basic exponential function formula, where a represents the starting amount and b represents the decay factor
$y = 20(1 - r)^x$	Replace b with the decay factor formula and substitute 20 for a.
$y = 20(1 - 0.03)^x$	Substitute 0.03 for r (rate of decay) as a decimal.
$y = 20(0.97)^x$	Subtract $1 - 0.03$.

The exponential decay model that represents this scenario is $y = 20(0.97)^x$.

5. Sketch the graph of the relationship.



6. Describe what the domain, the range, and the point (0, 20) represent.

The domain represents the time, in hours, that it takes for the mixture to evaporate, while the range represents the amount of the mixture, in kg, that is still present at a given time. The y-intercept of (0, 20) represents the starting amount, in kg, of the mixture.

Instruction

Example 2

A bank account has a starting balance of s dollars. It earns an annual interest rate, r, and is left untouched for n years. The account balance, b, can be calculated using the formula $b = s(1 + r)^n$. Create a graph and use it to determine the account balance to the nearest dollar if the account starts with \$100, has an annual interest rate of 5%, and is left untouched for the following number of years:

- 5 years
- 21 years
- 25.7 years

1. Substitute the known values into the formula.

   $b = 100(1 + 0.05)^n$ Replace 100 for s and 0.05 for r. Leave n and b as unknown variables.

2. Graph the equation in step 1 using x as the variable for n.

   On a TI-83/84:

   Step 1: Push [Y=] and type the expression $100(1 + 0.05)^x$ in Y_1.

   Step 2: Push [GRAPH].

   Step 3: Adjust the window as necessary by pushing ZOOM–ZoomFit, then ZOOM–Zoom Out if necessary until the entire graph is visible.

   Step 4: Push [2ND][GRAPH] to call up the TABLE screen, and identify the y-value when $x = 5$.

   On a TI-Nspire:

   Step 1: From the Graphs and Geometry Application, type $100(1 + 0.05)^x$ in *f1(x)*.

   Step 2: Push [menu] and select 4: Window to adjust the window, if necessary.

(*continued*)

Step 3: Select 2: View and 8: Add Function Table. The table will appear in a split screen so you can view the table and graph together.

Step 4: Use the up/down/left/right arrows to move within the table. Identify the *y*-value when $x = 5$.

After 5 years, the account balance will be \$127.63.

3. Repeat the calculator process used in step 2 to identify the *y*-value when $x = 21$.

   After 21 years, the account balance will be \$278.60.

4. Use the following calculator process to identify the *y*-value when $x = 25.7$.

   On a TI-83/84:

   Step 1: Push [Y=] and type the expression $100(1 + 0.05)^x$ in Y_1.

   Step 2: Push [GRAPH].

   Step 3: Adjust the window as necessary by pushing ZOOM–ZoomFit, then ZOOM–Zoom Out if necessary until the entire graph is visible.

   Step 4: Push [2ND][WINDOW] to call up the TABLE SETUP screen, and change Independent from AUTO to ASK.

   Step 5: Push [2ND][GRAPH] to call up the TABLE screen, and type in 25.7. Push [ENTER].

   On a TI-Nspire:

   Step 1: From the Graphs and Geometry Application, type $100(1 + 0.05)^x$ in *f1(x)*.

   Step 2: Push [menu] and select 4: Window to adjust the window, if necessary.

(*continued*)

Step 3: Select 2: View and 8: Add Function Table. The table will appear in a split screen so you can view the table and graph together.

Step 4: From within the table, push [menu] and select 5: Function Table.

Step 5: Select 3: Edit Function Table Settings and press [enter].

Step 6: Use TAB to move within this screen. Change Independent from AUTO to ASK. Press [ok].

Step 7: Type in 25.7. The *y*-value will automatically be displayed.

After 25.7 years, the account balance will be $350.40.

Example 3

A sports utility vehicle (SUV) was purchased for $25,000. According to an automobile dealers association, the buyer can expect the SUV to lose approximately 10% of its value annually. The function $f(x) = 25{,}000(0.9)^x$ describes the value of the SUV x years after its purchase.

- Does the situation described represent exponential growth or exponential decay, and how can you tell based on the function $f(x)$?
- What is the y-intercept of $f(x)$ and what does it represent?
- After how many years will the SUV be worth half its original price?
- In 8 years, how much could the owner expect to get for the SUV?

1. Determine whether the situation represents exponential growth or decay.

   The situation described represents exponential decay because the car is losing its value over time. The function $f(x)$ depicts this as well because the base of 0.9 is between 0 and 1, which is a characteristic of exponential decay functions.

2. Determine the y-intercept of $f(x)$ graphically and confirm algebraically.

On a TI-83/84:

Step 1: Push [Y=] and type the expression $25{,}000(0.9)^x$ in Y_1.

Step 2: Push [GRAPH].

Step 3: Adjust the window as necessary by pushing ZOOM–ZoomFit, then ZOOM–Zoom Out if necessary until the entire graph is visible.

Step 4: Push [2ND][GRAPH] to call up the TABLE screen, and identify the y-value when $x = 0$.

On a TI-Nspire:

Step 1: From the Graphs and Geometry Application, type $25{,}000(0.9)^x$ in *f1(x)*.

Step 2: Push [menu] and select 4: Window to adjust the window, if necessary.

Step 3: Select 2: View and 8: Add Function Table. The table will appear in a split screen so you can view the table and graph together.

Step 4: Use the up/down/left/right arrows to move within the table. Identify the y-value when $x = 0$.

Graphically, the y-intercept of $f(x)$ is 25,000. Next, let's confirm this algebraically.

$f(x) = 25{,}000(0.9)^x$	Original function
$f(x) = 25{,}000(0.9)^0$	Substitute 0 for x because the y-intercept represents the initial price of the SUV at time 0.
$f(x) = 25{,}000(1)$	A number raised to the 0 power is equal to 1.
$f(x) = 25{,}000$	Simplify.

Algebraically, the y-intercept of $f(x)$ is 25,000. The y-intercept represents the initial price of the SUV.

3. Determine when the SUV will be worth half its original price.

   Half of the original price is $12,500.

$f(x) = 25{,}000(0.9)^x$	Original function representing the value of the SUV x years after its purchase
$12{,}500 = 25{,}000(0.9)^x$	Substitute 12,500 for $f(x)$.

   Solve the equation for x by graphing the expressions on each side of the equal sign in the calculator and finding the point of intersection.

   On a TI-83/84:

   Step 1: Push [Y=] and type $25{,}000(0.9)^x$ in Y_1. Type 12,500 in Y_2.

   Step 2: Push [GRAPH] to graph both expressions on the same coordinate plane.

   Step 3: Adjust the window ZOOM–ZoomFit until the point of intersection is visible.

   Step 4: Push [2ND][TRACE] to call up the CALC screen, and scroll to 5: Intersect. Push [ENTER] to select the first graph, then push [ENTER] to select the second graph. Push [ENTER] again to find the point of intersection. The point of intersection is at $x = 6.58$, $y = 12{,}500$.

   Step 5: The x-value, 6.58, represents the solution to the equation.

   On a TI-Nspire:

   Step 1: From the Graphs and Geometry Application, type $25{,}000(0.9)^x$ in *f1(x)* and type 12,500 in *f2(x)*.

   Step 2: Push [menu] and select 4: Window to adjust the window until the point of intersection is visible, if necessary.

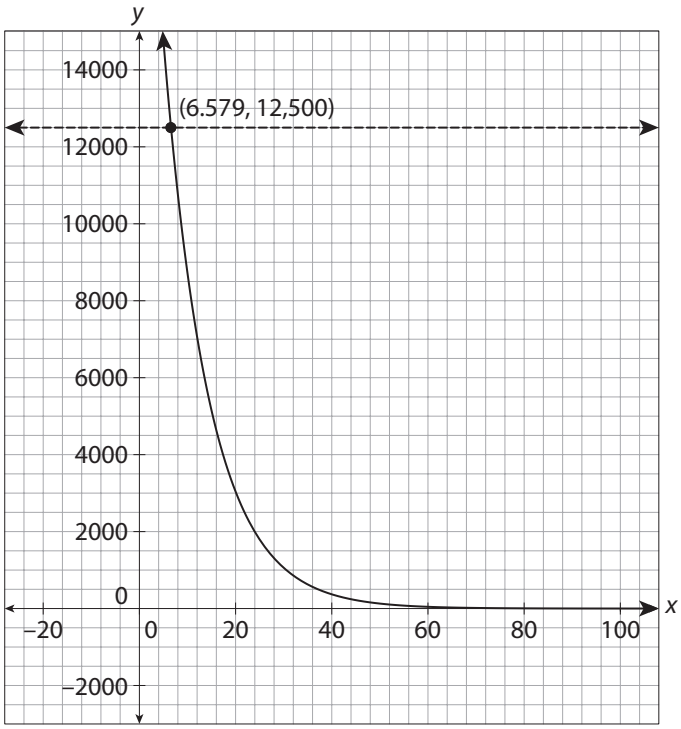
   Step 3: Leaving the graph window the same, push [menu] and select 6: Points & Lines. Then select 3: Intersection Points.

(*continued*)

Step 4: When the pointing hand appears, click on each graph and all intersection points will be displayed in the viewing window. The point of intersection is at $x = 6.58$, $y = 12{,}500$.

Step 5: The x-value of the point of intersection, 6.58, represents the solution to the equation.

The resulting graph should resemble the following:



After approximately 6.58 years, the SUV will be worth half its original price.

Instruction

4. Determine the value of the SUV after 8 years.

On a TI-83/84:

Step 1: Push [Y=] and type the expression $25{,}000(0.9)^x$ in Y_1.

Step 2: Push [GRAPH].

Step 3: Adjust the window as necessary by pushing ZOOM–ZoomFit, then ZOOM–Zoom Out if necessary until the entire graph is visible.

Step 4: Push [2ND][GRAPH] to call up the TABLE screen, and identify the y-value when $x = 8$.

On a TI-Nspire:

Step 1: From the Graphs and Geometry Application, type $25{,}000(0.9)^x$ in *f1(x)*.

Step 2: Push [menu] and select 4: Window to adjust the window, if necessary.

Step 3: Select 2: View and 8: Add Function Table. The table will appear in a split screen so you can view the table and graph together.

Step 4: Use the up/down/left/right arrows to move within the table. Identify the y-value when $x = 8$.

After 8 years, the owner could expect to sell the car for $10,762.