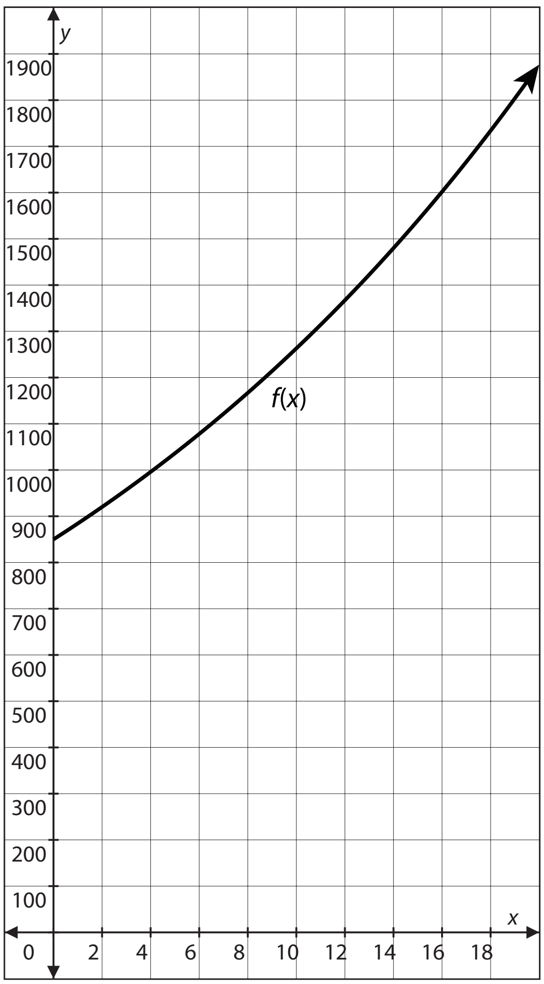
Guided Practice 4.7

Example 1

Compare the properties of each of the following two functions over the interval [0, 16].

Function A



Function B

x	*g*(*x*)
0	850
4	976.55
8	1121.94
12	1288.98
16	1480.88

1. Compare the y-intercepts of each function.

   Identify the y-intercept of the graphed function, $f(x)$.

   The graphed function appears to cross the y-axis at the point (0, 850).

   According to the table, $g(x)$ has a y-intercept of (0, 850).

   Both functions have a y-intercept of (0, 850).

2. Compare the rate of change for each function over the interval [0, 16].

   Calculate the rate of change over the interval [0, 16] for $f(x)$.

   Let $(x_1, y_1) = (0, 850)$.

   Determine (x_2, y_2) from the graph.

   The value of y when x is 16 is approximately 1,600.

   Let $(x_2, y_2) = (16, 1600)$.

   Calculate the rate of change using the slope formula.

$m=\dfrac{y_2-y_1}{x_2-x_1}$	Slope formula
$=\dfrac{(1600)-(850)}{(16)-(0)}$	Substitute (0, 850) for (x_1, y_1) and (16, 1600) for (x_2, y_2).
$=\dfrac{750}{16}$	Simplify.
$= 46.875$	

   The rate of change for $f(x)$ is approximately 47.

   Calculate the rate of change over the interval [0, 16] for $g(x)$.

   Let $(x_1, y_1) = (0, 850)$.

   Determine (x_2, y_2) from the table.

   The value of y when x is 16 is 1,480.88.

   Let $(x_2, y_2) = (16, 1480.88)$.

(*continued*)

Calculate the rate of change using the slope formula.

$m=\dfrac{y_2-y_1}{x_2-x_1}$	Slope formula
$=\dfrac{(1480.88)-(850)}{(16)-(0)}$	Substitute (0, 850) for (x_1, y_1) and (16, 1480.88) for (x_2, y_2).
$=\dfrac{630.88}{16}$	Simplify.
$= 39.43$	

The rate of change for $g(x)$ is 39.43.

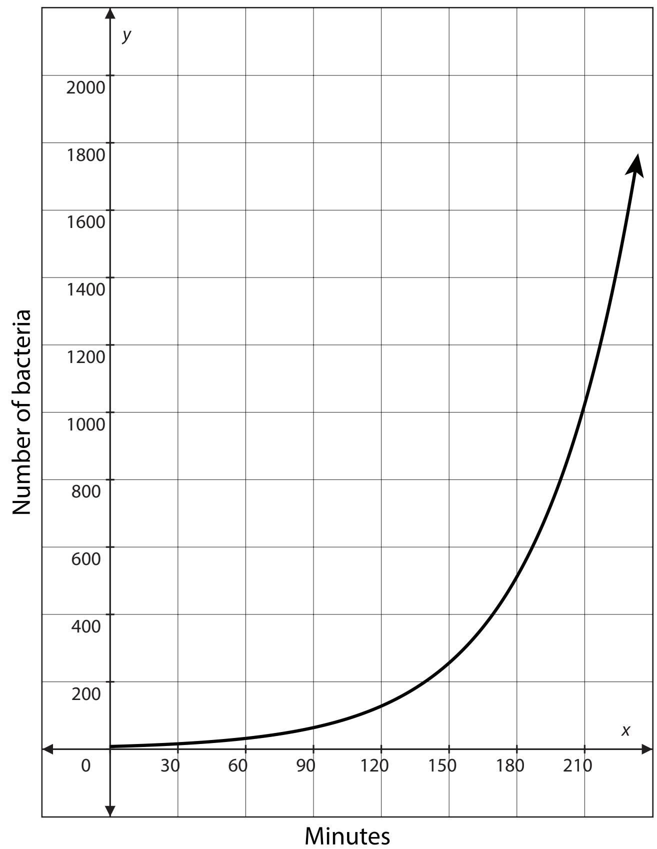
The rate of change for the graphed function, $f(x)$, is greater over the interval [0, 16] than the rate of change for the function in the table, $g(x)$.

3. Summarize your findings.

   The y-intercepts of both functions are the same; however, the graphed function, $f(x)$, has a greater rate of change over the interval [0, 16].

Example 2

A Petri dish started with a population of 8 bacteria. The population doubles every 15 minutes. A second population of bacteria, shown in the following graph, also started with 8 bacteria. Compare the properties of the functions that represent each population over the interval [150, 210].



1. Compare the y-intercepts of each function.

   According to the scenario, the initial number of bacteria for both functions is 8; therefore, the y-intercept is 8.

2. Compare the rate of change for each function over the interval [150, 210].

Calculate the rate of change over the interval [150, 210] for the graphed function.

Determine (x_1, y_1) from the graph.

The value of y when x is 150 is approximately 275.

Let $(x_1, y_1) = (150, 275)$.

Determine (x_2, y_2) from the graph.

The value of y when x is 210 is approximately 1,000.

Let $(x_2, y_2) = (210, 1000)$.

Calculate the rate of change using the slope formula.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \dfrac{(1000) - (275)}{(210) - (150)}$ Substitute (150, 275) for (x_1, y_1) and (210, 1000) for (x_2, y_2).

$= \dfrac{725}{60}$ Simplify.

≈ 12

The rate of change for the graphed function is approximately 12 bacteria per minute.

To determine the rate of change for the other function described in the scenario, first write a function rule to represent the situation.

$$f(x) = 8(2)^{\frac{x}{15}}$$

Determine the value for y when x is 150 using the function.

$f(x) = 8(2)^{\frac{x}{15}}$ Original function

$f(x) = 8(2)^{\frac{(150)}{15}}$ Substitute 150 for x.

$f(150) = 8(2)^{10}$ Simplify.

(*continued*)

$f(150) = 8(1024)$

$f(150) = 8192$

$(x_1, y_1) = (150, 8192)$

Determine the value for y when x is 210 using the function.

$f(x)=8(2)^{\frac{x}{15}}$	Original function
$f(x)=8(2)^{\frac{(210)}{15}}$	Substitute 210 for x.
$f(210) = 8(2)^{14}$	Simplify as needed.

$f(210) = 8(16,384)$

$f(210) = 131,072$

$(x_2, y_2) = (210, 131,072)$

Calculate the rate of change using the slope formula.

$m=\dfrac{y_2-y_1}{x_2-x_1}$	Slope formula
$=\dfrac{(131,072)-(8192)}{(210)-(150)}$	Substitute (150, 8192) for (x_1, y_1) and (210, 131,072) for (x_2, y_2).
$=\dfrac{122,880}{60}$	Simplify as needed.

$= 2048$

The rate of change for the function in the table is 2,048 bacteria per minute.

The rate of change for the graphed function is less steep over the interval [150, 210] than the rate of change for the other function.

3. Summarize your findings.

The y-intercepts of both functions are the same; however, the graphed function is less steep over the interval [150, 210]. The population of bacteria shown by the graphed function are doubling at a slower rate than the bacteria in the first function described.

Example 3

A pendulum swings to 90% of its previous height. Pendulum A starts at a height of 50 centimeters. Its height at each swing is modeled by the function $f(x) = 50(0.90)^x$. The height after every fifth swing of Pendulum B is recorded in the following table. Compare the properties of each function over the interval [5, 15].

x	$g(x)$
0	100
5	59.05
10	34.87
15	20.59
20	12.16

1. Compare the y-intercepts of each function.

   Identify the y-intercept of Pendulum A.

   The problem states that the pendulum starts at a height of 50 centimeters.

   The y-intercept of the function is (0, 50).

   Identify the y-intercept of Pendulum B.

   The value of $g(x)$ is 100 when x is 0.

   The y-intercept of the function is 100.

2. Compare the rate of change for each function over the interval [5, 15].

Calculate the rate of change over the interval [5, 15] for Pendulum A.

Determine (x_1, y_1) from the function.

$f(x) = 50(0.90)^x$	Original function
$f(5) = 50(0.90)^{(5)}$	Substitute 5 for x.
$f(5) = 29.52$	Simplify.

Let $(x_1, y_1) = (5, 29.52)$.

Determine (x_2, y_2) from the function.

$f(x) = 50(0.90)^x$	Original function
$f(15) = 50(0.90)^{(15)}$	Substitute 15 for x.
$f(15) \approx 10.29$	Simplify.

The value of y when x is 15 is approximately 10.29.

Let $(x_2, y_2) = (15, 10.29)$.

Calculate the rate of change using the slope formula.

$m=\dfrac{y_2-y_1}{x_2-x_1}$	Slope formula
$=\dfrac{(10.29)-(29.52)}{(15)-(5)}$	Substitute (5, 29.52) for (x_1, y_1) and (15, 10.29) for (x_2, y_2).
$=\dfrac{-19.23}{10}=-1.923$	Simplify.

The rate of change for Pendulum A's function is approximately –1.923 centimeters per swing.

(*continued*)

Calculate the rate of change over the interval [5, 15] for Pendulum B.

Let $(x_1, y_1) = (5, 59.05)$.

Let $(x_2, y_2) = (15, 20.59)$.

Calculate the rate of change using the slope formula.

$m=\dfrac{y_2-y_1}{x_2-x_1}$	Slope formula
$=\dfrac{(20.59)-(59.05)}{(15)-(5)}$	Substitute (5, 59.05) for (x_1, y_1) and (15, 20.59) for (x_2, y_2).
$=\dfrac{-38.46}{10}=-3.846$	Simplify.

The rate of change for Pendulum B's function is approximately –3.846 centimeters per swing.

The rate of change for Pendulum B is greater over the interval [5, 15] than the rate of change for Pendulum A.

3. Summarize your findings.

The *y*-intercept of Pendulum A is less than the *y*-intercept of Pendulum B. This means that Pendulum B begins higher than Pendulum A. The rate of change for Pendulum A is less than the rate of change for Pendulum B. This means that Pendulum B is losing height faster than Pendulum A.