Guided Practice 4.8

Example 1

A school tracks the total number of students enrolled each year. The school uses the change in the total number of students to estimate how many students have been enrolled in the school each year since 2000. If t is the number of years after 2000, the total number of students, $f(t)$, can be estimated using the function $f(t) = 250(0.98)^t$. How is the total number of students changing each year?

1. Identify the yearly rate of change in the function.

   In the function $f(t) = 250(0.98)^t$, $b = 0.98$, and $0 < b < 1$. Therefore, the function represents exponential decay, which can be modeled by the form $f(t) = a(1 - r)^t$. To find the yearly rate of change, r, set b equal to $1 - r$ and solve for r.

   $0.98 = 1 - r$

   $0.98 - 1 = -r$

   $-0.02 = -r$

   $0.02 = r$

   The rate of change is 0.02. In this decay model, this indicates a decrease of 2% per year.

2. Describe how the rate of change relates to the change of the dependent quantity.

   The rate of change explains how the dependent quantity is growing or decaying over the given unit of time. The dependent quantity is the number of students, and the unit of time is "each year." The function $f(t) = 250(0.98)^t$ shows exponential decay, and the rate of change is 0.02, or 2%. The independent variable t is the time in years. The total number of students is decreasing by 2% each year.

Example 2

A bank offers a savings account with interest that is compounded monthly. In other words, the interest earned is added to the account every month instead of once a year. Dillon opened a savings account with $500. If t is the time in years the account has been open, the balance in his account, $f(t)$, is $f(t) = 500(1.004)^{12t}$. What is the estimated yearly exponential growth rate? Describe how this rate relates to the yearly change of the balance in Dillon's account.

1. Use the Power of a Power Property to simplify the exponential expression.

   The expression $(1.004)^{12t}$ can be simplified using the Power of a Power Property to remove the factor of the exponent.

$f(t) = 500(1.004)^{12t}$	Original function
$f(t) = 500\left(1.004^{12}\right)^t$	Apply the Power of a Power Property.
$f(t) = 500(1.049)^t$	Simplify.

2. Identify the yearly rate of change in the function.

   In the function $f(t) = 500(1.049)^t$, $b = 1.049$ and $b > 1$. Therefore, the function represents exponential growth, which can be modeled by the form $f(t) = a(1 + r)^t$. To find the yearly rate of change, r, set b equal to $1 + r$ and solve for r.

   $1.049 = 1 + r$

   $1.049 - 1 = r$

   $0.049 = r$

   The rate of change is 0.049, or an increase of 4.9% per year.

3. Describe how the rate of change relates to the change of the dependent quantity.

   The rate of change explains how the dependent quantity is growing or decaying over the given unit of time. The yearly rate of change in the function $f(t) = 500(1.004)^{12t}$ is 4.9%. The independent variable t is the time in years. The total balance of the account, in dollars, is increasing by approximately 4.9% each year.

Example 3

A number of bacteria, $f(t)$, at any time t, in hours, can be estimated using the function $f(t) = 3000(1.24)^t$. What was the initial size of the bacteria colony? Is the bacteria population exponentially decaying or growing?

1. Identify the value of the dependent quantity at $t = 0$.

   In an exponential function of the form $f(t) = a(1 + r)^t$, a is the value of the dependent quantity, $f(t)$, when $t = 0$. In the function $f(t) = 3000(1.24)^t$, the factor a is 3,000. Recall that any number raised to a power of 0 is equal to 1. Therefore, the initial size of the bacteria colony was 3,000 bacteria.

2. Identify the hourly rate of change in the function.

   In the function $f(t) = 3000(1.24)^t$, $b = 1.24$ and $b > 0$. Therefore, there is exponential growth, which can be modeled by the form $f(t) = a(1 + r)^t$. To find the yearly rate of change, r, set b equal to $1 + r$ and solve for r.

   $1.24 = 1 + r$

   $1.24 - 1 = r$

   $0.24 = r$

   The rate of change is 0.24, or an increase of 24% per hour.

3. Describe how the rate of change relates to the change of the dependent quantity.

   The rate of change explains how the dependent quantity is growing or decaying over the given unit of time. The hourly rate of change in the function $f(t) = 3000(1.24)^t$ is 24%. The independent variable t is the time in hours. The total number of bacteria in the colony is increasing by approximately 24% every hour, so the bacteria population is exponentially growing.