Instruction

## Guided Practice 4.9

### Example 1

The following table illustrates the relationship between the solubility of potassium nitrate and the temperature of water it is placed in.

| **Temperature in degrees Celsius ($t$)** | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 |
|---|---|---|---|---|---|---|---|---|
| **Solubility in grams per 100 grams of water $f(t)$** | 15 | 22 | 32 | 48 | 65 | 85 | 108 | 137 |

Determine an exponential regression equation that best fits the data presented in the table. Use this equation to determine how many grams of potassium nitrate will dissolve in water heated to 100° C. In order for 90 grams of potassium nitrate to be soluble, to what temperature should the water be heated?

1. Use the calculator to create a model for the data.

**On a TI-83/84:**

Step 1: Press [STAT][EDIT] and select 1: Edit. Enter $x$-values in $L_1$ of the table and $y$-values in $L_2$.

Step 2: Press [STAT][CALC] and select 0: ExpReg. Press Calculate, and the calculator will calculate the coefficient and rate of change for the exponential equation.

Step 3: Substitute the coefficient and rate of change into the general form of the exponential equation.

**On a TI-Nspire:**

Step 1: Enter data on the List and Spreadsheet App, with the $x$-coordinates in List A and the $y$-coordinates in List B. Be sure to name your lists as $x$ for List A and $y$ for List B.

Step 2: From the [home] menu, choose 5: Data and Statistics. Press [enter].

Step 3: Using the NavPad, move to the bottom and choose the $x$-variable list name. Then move to the left and choose the $y$-variable list name to create the scatter plot.

(*continued*)

Step 4: To determine the best-fit exponential regression equation, press [menu], select 3: Actions, then select 5: Regression. Next, select A: Exponential Regression. The curve and equation will appear on the screen.

The exponential regression equation created based on the data in the table is $y = 16.577(1.032)^x$.

2. Determine how many grams of potassium nitrate will dissolve in water heated to 100° C.

| | |
|---|---|
| $y = 16.577(1.032)^x$ | Regression model |
| $y = 16.577(1.032)^{100}$ | Substitute 100 for $x$. |
| $y = 386.79$ | Simplify. |

According to the regression model, approximately 386.79 grams of potassium nitrate will dissolve if the water is heated to 100° C.

3. Determine the temperature of the water that will dissolve 90 grams of potassium nitrate.

| | |
|---|---|
| $y = 16.577(1.032)^x$ | Regression model |
| $90 = 16.577(1.032)^x$ | Substitute 90 for $y$. |

Solve the equation for $x$ by graphing the expressions on each side of the equal sign in the calculator and finding the point of intersection.

**On a TI-83/84:**

Step 1: Press [Y=] and type $16.577(1.032)^x$ in $Y_1$. Type 90 in $Y_2$.

Step 2: Press [GRAPH] to graph both expressions on the same coordinate plane.

Step 3: Adjust the window ZOOM–ZoomFit until the point of intersection is visible.

Step 4: Press [2ND][TRACE][CALC], and scroll to 5: Intersect. Press [ENTER] to select the first graph, then press [ENTER] again to select the second graph. Press [ENTER] a third time to find the point of intersection. The point of intersection is at $x = 53.71$, $y = 90$.

Step 5: The $x$-value, 53.71, represents the solution to the equation.

(*continued*)

**On a TI-Nspire:**

Step 1: From the Graphs and Geometry Application, type $16.577(1.032)^x$ in $f1(x)$ and type 90 in $f2(x)$.

Step 2: Press [menu] and select 4: Window to adjust the window until the point of intersection is visible, if necessary.

Step 3: Leaving the graph window the same, press [menu] and select 6: Points & Lines. Then select 3: Intersection Points.

Step 4: When the pointing hand appears, click on each graph, and all intersection points will be displayed in the viewing window. The point of intersection is at $x = 53.71$, $y = 90$.

Step 5: The $x$-value of the point of intersection, 53.71, represents the solution to the equation.

If the water is heated to approximately 53.71° C, 90 grams of potassium nitrate will be soluble.

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**Example 2**

The number of users on Facebook between 2004 and 2012 is represented in the following table.

| **Year (*t*)** | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
|---|---|---|---|---|---|---|---|---|---|
| **Number of users in millions *f*(*t*)** | 1 | 5.5 | 12 | 50 | 100 | 350 | 550 | 800 | 1000 |

Determine an exponential regression equation that best fits the data, assuming $x$ represents the years after 2004. Use the equation to determine how many Facebook users were predicted to exist in 2015 based on the model. Compare your answer to the actual number of Facebook users in 2015.

Is the equation you found a good model for finding the actual number of Facebook users? If not, what are some possible reasons that may explain the discrepancy between the values found using the model and those that actually occurred?

According to the model you found, determine the approximate year the number of Facebook users is expected to reach 10 billion (10,000 million).

1. Use the calculator to create a model for the data.

   **On a TI-83/84:**

   Step 1: Press [STAT][EDIT] and select 1: Edit. Enter $x$-values beginning at 0 in $L_1$ of the table and $y$-values in $L_2$.

   Step 2: Press [STAT][CALC] and select 0: ExpReg. Press Calculate, and the calculator will calculate the coefficient and rate of change for the exponential equation.

   Step 3: Substitute the coefficient and rate of change into the general form of the exponential equation.

   **On a TI-Nspire:**

   Step 1: Enter data on the List and Spreadsheet App, with the $x$-coordinates in List A and the $y$-coordinates in List B. Be sure to name your lists as $x$ for List A and $y$ for List B.

   Step 2: From the [home] menu, choose 5: Data and Statistics. Press [enter].

   Step 3: Using the NavPad, move to the bottom and choose the $x$-variable list name. Then move to the left and choose the $y$-variable list name to create the scatter plot.

(*continued*)

Step 4: To determine the best-fit exponential regression equation, press [menu], select 3: Actions, then select 5: Regressions. Next, select A: Exponential Regression. The curve and equation will appear on the screen.

The exponential regression equation created based on the data in the table is $y = 2.217(2.386)^x$, where $x$ represents the number of years after 2004.

2. Determine how many Facebook users were predicted to exist in 2015 based on the model we found.

| | |
|---|---|
| $x = 11$ | 2015 is 11 years after 2004 |
| $y = 2.217(2.386)^{11}$ | Substitute 11 for $x$. |
| $y = 31.633$ billion | Simplify. |

According to the model we found, approximately 31 billion Facebook users were expected to exist in 2015. The actual number of monthly active Facebook users in 2015 was closer to 1.4 billion.

There are many possible reasons there would be a difference between the actual number of users and the predicted number of users based on the model. Some possible reasons include taking into consideration groups of people around the world who still have limited or no access to a computer or mobile device. People also delete their Facebook accounts for various reasons, even though they may have been active users at one point.

3. Determine the approximate year the number of Facebook users is expected to reach 10 billion.

| | |
|---|---|
| $y = 2.217(2.386)^x$ | Regression model |
| $10{,}000 = 2.217(2.386)^x$ | Substitute 10,000 million (equivalent to 10 billion) for $y$. |

Solve the equation for $x$ by graphing the expressions on each side of the equal sign in the calculator and finding the point of intersection.

**On a TI-83/84:**

Step 1: Press [Y=] and type $2.217(2.386)^x$ in $Y_1$. Type 10,000 in $Y_2$.

Step 2: Press [GRAPH] to graph both expressions on the same coordinate plane.

Step 3: Adjust the window ZOOM–ZoomFit until the point of intersection is visible.

Step 4: Press [2ND][TRACE][CALC], and scroll to 5: Intersect. Press [ENTER] to select the first graph, then press [ENTER] again to select the second graph. Press [ENTER] a third time to find the point of intersection. The point of intersection is at $x = 9.68$, $y = 10{,}000$.

Step 5: The $x$-value, 9.68, represents the solution to the equation.

**On a TI-Nspire:**

Step 1: From the Graphs and Geometry Application, type $2.217(2.386)^x$ in $f1(x)$ and type 10,000 in $f2(x)$.

Step 2: Press [menu] and select 4: Window to adjust until the point of intersection is visible, if necessary.

Step 3: Leaving the graph window the same, press [menu], select 6: Points & Lines, then select 3: Intersection Points.

Step 4: When the pointing hand appears, click on each graph, and all intersection points will be displayed in the viewing window. The point of intersection is at $x = 9.68$, $y = 10{,}000$.

Step 5: The $x$-value of the point of intersection, 9.68, represents the solution to the equation.

According to the model, the number of Facebook users should have reached 10 billion approximately 9.68 years after 2004, or in the year 2013.

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**Example 3**

You are interested in finding out how much money a restaurant server makes in tips based on the cost of the meals that customers purchase. The following table represents the data you collect on the amount of tips earned for various meal prices.

| **Meal cost ($c$)** | \$4.75 | \$6.82 | \$12.52 | \$20.42 | \$8.97 |
|---|---|---|---|---|---|
| **Tip received $f(c)$** | \$0.50 | \$0.90 | \$1.50 | \$3.00 | \$1.00 |

Determine an exponential regression equation that best fits the data presented in the table, and use the equation to determine how much a server is expected to make on a meal that costs \$30. Use your model to determine the price of a meal a customer would have to purchase for a server to earn at least a \$5.00 tip.

1. Use the calculator to create a model for the data.

**On a TI-83/84:**

Step 1: Press [STAT][EDIT] and select 1: Edit. Enter $x$-values in $L_1$ of the table and $y$-values in $L_2$.

Step 2: Press [STAT][CALC] and select 0: ExpReg. Press Calculate, and the calculator will calculate the coefficient and rate of change for the exponential equation.

Step 3: Substitute the coefficient and rate of change into the general form of the exponential equation.

**On a TI-Nspire:**

Step 1: Enter data on the List and Spreadsheet App, with the $x$-coordinates in List A and the $y$-coordinates in List B. Be sure to name your lists as $x$ for List A and $y$ for List B.

Step 2: From the [home] menu, choose 5: Data and Statistics. Press [enter].

Step 3: Using the NavPad, move to the bottom and choose the $x$-variable list name. Then move to the left and choose the $y$-variable list name to create the scatter plot.

Step 4: To determine the best-fit exponential regression equation, press [menu], select 3: Actions, then select 5: Regression. Next, select A: Exponential Regression. The curve and equation will appear on the screen.

The exponential regression equation created based on the data in the table is $y = 0.3727(1.111)^x$.

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2. Determine how much a server is expected to make on a meal that costs $30.

| | |
|---|---|
| $y = 0.3727(1.111)^x$ | Regression model |
| $y = 0.3727(1.111)^{30}$ | Substitute 30 for $x$. |
| $y = 8.776559$ | Simplify. |

According to the model, a server is expected to make approximately $8.77 in tips if a customer purchases a $30.00 meal.

3. Determine the price of a meal a customer would have to purchase for a server to earn at least a $5.00 tip.

| | |
|---|---|
| $y = 0.3727(1.111)^x$ | Regression model |
| $5 = 0.3727(1.111)^x$ | Substitute 5 for $y$. |

Solve the equation for $x$ by graphing the expressions on each side of the equal sign in the calculator and finding the point of intersection.

**On a TI-83/84:**

Step 1: Press [Y=] and type $0.3727(1.111)^x$ in $Y_1$. Type 5 in $Y_2$.

Step 2: Press [GRAPH] to graph both expressions on the same coordinate plane.

Step 3: Adjust the window ZOOM–ZoomFit until the point of intersection is visible.

Step 4: Press [2ND][TRACE][CALC], and scroll to 5: Intersect. Press [ENTER] to select the first graph, then press [ENTER] again to select the second graph. Press [ENTER] a third time to find the point of intersection. The point of intersection is at $x = 24.67$, $y = 5$.

Step 5: The $x$-value, 24.67, represents the solution to the equation.

(*continued*)

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**On a TI-Nspire:**

Step 1: From the Graphs and Geometry Application, type $0.3727(1.111)^x$ in $f1(x)$ and type 5 in $f2(x)$.

Step 2: Press [menu] and select 4: Window to adjust until the point of intersection is visible, if necessary.

Step 3: Leaving the graph window the same, press [menu], select t6: Points & Lines, then select 3: Intersection Points.

Step 4: When the pointing hand appears, click on each graph, and all intersection points will be displayed in the viewing window. The point of intersection is at $x = 24.67$, $y = 5$.

Step 5: The $x$-value of the point of intersection, 24.67, represents the solution to the equation.

According to the model, a server will earn at least \$5 in tips if a customer purchases a meal that costs at least \$24.67.