Guided Practice 5.3

Example 1

Factor the polynomial $12x^3 + 30x^2 + 42x$ by finding the greatest common factor (GCF). Verify your results using the Distributive Property.

1. Determine the common factors of the terms' coefficients and variables.

   First factor the coefficients (12, 30, and 42) into their prime factors.

   The prime factors are as follows:

   $12 = 2 \bullet 2 \bullet 3$

   $30 = 2 \bullet 3 \bullet 5$

   $42 = 2 \bullet 3 \bullet 7$

   Each coefficient's factors include 2 and 3, so $2 \bullet 3 = 6$ is common to all three terms.

   Next, determine the greatest power of x common to all three terms. The first term has x^3, the second term has x^2, and the third term has x (or x^1). Expand these terms:

   $x^3 = x \bullet x \bullet x$

   $x^2 = x \bullet x$

   $x^1 = x$

   Only x (or x to the first power) is common to all three terms. Combining this with the greatest numerical factor of 6, the resulting GCF is $6x$.

2. Write expressions for the remaining non-common factors.

   Write out the prime factors of each term:

   $12x^3 = 2 \bullet 2 \bullet 3 \bullet x \bullet x \bullet x$

   $30x^2 = 2 \bullet 3 \bullet 5 \bullet x \bullet x$

   $42x = 2 \bullet 3 \bullet 7 \bullet x$

   Eliminate the common factors of each term and determine which non-common factors are left:

   $12x^3 = \cancel{2} \bullet \boxed{2} \bullet \cancel{3} \bullet \cancel{x} \bullet \boxed{x} \bullet \boxed{x}$; the remaining factors are $2 \bullet x \bullet x$, or $2x^2$

   $30x^2 = \cancel{2} \bullet \cancel{3} \bullet \boxed{5} \bullet \cancel{x} \bullet \boxed{x}$; the remaining factors are $5 \bullet x$, or $5x$

   $42x = \cancel{2} \bullet \cancel{3} \bullet \boxed{7} \bullet \cancel{x}$; the remaining factor is 7

   The expressions for the remaining non-common factors are $2x^2$, $5x$, and 7.

3. Factor the original polynomial by writing it as the product of the GCF and a polynomial whose terms are the expressions found in the previous step.

   The GCF is $6x$. The remaining terms are $2x^2$, $5x$, and 7, which can be summed to produce the polynomial $2x^2 + 5x + 7$.

   The product of the GCF and this polynomial is $6x(2x^2 + 5x + 7)$.

4. Verify the result using the Distributive Property.

   To verify the result, apply the Distributive Property by multiplying $6x$ by each term of the polynomial.

   $6x(2x^2 + 5x + 7)$

   $= 6x \bullet 2x^2 + 6x \bullet 5x + 6x \bullet 7$

   $= 12x^3 + 30x^2 + 42x$

   The result of applying the Distributive Property is the original expression, $12x^3 + 30x^2 – 42x$. Thus, the polynomial is factored correctly and $12x^3 + 30x^2 + 42x = 6x(2x^2 + 5x + 7)$.

Example 2

Factor the polynomial $35xy^4 - 14x^4y^2z + 56x^3y^3z^3$ by finding the GCF. Verify your results using the Distributive Property.

1. Determine the common factors of the coefficients for each term.

   First factor the coefficients (35, –14, and 56) into their prime factors:

   $35 = 5 \bullet 7$

   $-14 = -1 \bullet 2 \bullet 7$

   $56 = 2 \bullet 2 \bullet 2 \bullet 7$

   Note that a negative number, like –14, will always have a negative factor that can be represented by –1.

   The only numerical factor that is common to all three terms is 7.

2. Determine the greatest power of each variable common to all terms, and multiply this by the common factor -from step 1 to complete the GCF.

   The first term has the variables xy^4, the second has variables x^4y^2z, and the third has variables $x^3y^3z^3$. Expand these terms:

   $xy^4 = x \bullet y \bullet y \bullet y \bullet y$

   $x^4y^2z = x \bullet x \bullet x \bullet x \bullet y \bullet y \bullet z$

   $x^3y^3z^3 = x \bullet x \bullet x \bullet y \bullet y \bullet y \bullet z \bullet z \bullet z$

   One factor of x and two factors of y, or y^2, are common to all three terms. Since z is not a factor of the first term, it is not a common factor.

   Multiplying these common variables by the result of step 1 yields a GCF of $7xy^2$.

3. Write expressions for the remaining non-common factors.

   Write out the prime factors of each term:

   $35xy^4 = 5 \bullet 7 \bullet x \bullet y \bullet y \bullet y \bullet y$

   $-14x^4y^2z = -1 \bullet 2 \bullet 7 \bullet x \bullet x \bullet x \bullet x \bullet y \bullet y \bullet z$

   $56x^3y^3z^3 = 2 \bullet 2 \bullet 2 \bullet 7 \bullet x \bullet x \bullet x \bullet y \bullet y \bullet y \bullet z \bullet z \bullet z$

   Eliminate the common factors of each term and determine which non-common factors are left:

   $35xy^4 = \boxed{5} \bullet \cancel{7} \bullet \cancel{x} \bullet \cancel{y} \bullet \cancel{y} \bullet \boxed{y} \bullet \boxed{y}$

   The remaining factors are $5 \bullet y \bullet y$, or $5y^2$.

   $-14x^4y^2z = \boxed{-1} \bullet \boxed{2} \bullet \cancel{7} \bullet \cancel{x} \bullet \boxed{x} \bullet \boxed{x} \bullet \boxed{x} \bullet \cancel{y} \bullet \cancel{y} \bullet \boxed{z}$

   The remaining factors are $-1 \bullet 2 \bullet x \bullet x \bullet x \bullet z$, which simplifies to $-2x^3z$.

   $56x^3y^3z^3 = \boxed{2} \bullet \boxed{2} \bullet \boxed{2} \bullet \cancel{7} \bullet \cancel{x} \bullet \boxed{x} \bullet \boxed{x} \bullet \cancel{y} \bullet \cancel{y} \bullet \boxed{y} \bullet \boxed{z} \bullet \boxed{z} \bullet \boxed{z}$

   The remaining factors are $2 \bullet 2 \bullet 2 \bullet x \bullet x \bullet y \bullet z \bullet z \bullet z$, which simplifies to $8x^2yz^3$.

   The expressions for the remaining non-common factors are $5y^2$, $-2x^3z$, and $8x^2yz^3$.

4. Factor the original polynomial by writing it as the product of the GCF and a polynomial whose terms are the expressions found in the previous step.

   The GCF is $7xy^2$. The remaining terms are $5y^2$, $-2x^3z$, and $8x^2yz^3$, which can be summed to produce the polynomial $5y^2 - 2x^3z + 8x^2yz^3$.

   The product of the GCF and this polynomial is $7xy^2(5y^2 - 2x^3z + 8x^2yz^3)$.

5. Verify the result using the Distributive Property.

   Apply the Distributive Property by multiplying $7xy^2$ by each term of the polynomial.

   $7xy^2(5y^2 - 2x^3z + 8x^2yz^3)$

   $= 7xy^2 \bullet 5y^2 - 7xy^2 \bullet 2x^3z + 7xy^2 \bullet 8x^2yz^3)$

   $= 35xy^4 - 14x^4y^2z + 56x^3y^3z^3$

   The result of applying the Distributive Property is the original expression, $35xy^4 - 14x^4y^2z + 56x^3y^3z^3$. Thus, the polynomial is factored correctly and $35xy^4 - 14x^4y^2z + 56x^3y^3z^3 = 7xy^2(5y^2 - 2x^3z + 8x^2yz^3)$.

Example 3

The polynomial $15x^2 - 3x$ represents the area of a rectangular garden plot in square yards, where the length of the garden is equal to the GCF. Determine expressions for the length and width of the garden.

1. Determine the common factors of the terms' coefficients.

   First factor the coefficients (15 and –3) into their prime factors:

   $15 = 3 \bullet 5$

   $-3 = -1 \bullet 3$

   The only numerical factor that is common to both terms is 3.

2. Determine the greatest power of each variable common to all terms, and multiply it by the common factor from step 1 to complete the GCF.

   The first term has the variable x^2, or $x \bullet x$.

   The second term has only x, or x^1. Only x, or x to the first power, is common to both terms.

   Multiplying by the result from step 1 gives a GCF of $3x$.

3. Write expressions for the remaining non-common factors.

   The original first term, $15x^2$, has factors of 3, 5, x, and x, of which 3 and x are common. The expression from the remaining factors is $5 \bullet x$, or $5x$.

   The original second term, $-3x$, has factors of –1, 3, and x. Ignoring the common factors of 3 and x, the only non-common factor is –1.

4. Write the polynomial whose terms are the expressions found in step 3.

   The terms found in step 3 are $5x$ and –1; therefore, the polynomial is $(5x - 1)$.

5. Identify the required expressions for the length and width.

   The original problem stated that the length of the garden is equal to the GCF, which we found in step 2 to be $3x$. Therefore, the length is $3x$.

   To find the width, use the area formula: $A = lw$.

   The area of the garden in square yards is $15x^2 - 3x$, the original polynomial, and the garden's length is $3x$, so $15x^2 - 3x = 3x \bullet w$.

   The GCF of $3x$ was found by factoring the area into $3x(5x - 1)$.

   Therefore, $(5x - 1)$ is the width of the garden.

   The length of the garden is $3x$ yards, and the width is $(5x - 1)$ yards.