Guided Practice 5.4

Example 1

Factor $9x^2 - 16$, and then verify your results.

1. Determine any common factors of the given binomial, if common factors exist.

   The prime factors of $9x^2$ are 3, 3, x, and x.

   The prime factors of 16 are 2, 2, 2, and 2.

   There are no factors that are common to both terms.

2. Determine if the given binomial meets the conditions of the difference of two squares.

   Binomials that are the difference of two squares are of the form $x^2 - y^2$.

   In other words, both terms must be perfect squares, and the second term must be subtracted from the first term.

   $9x^2$ is a perfect square because $9x^2$ is the product of two equal factors ($3x \bullet 3x = 9x^2$).

   16 is also a perfect square because 16 is the product of two equal factors ($4 \bullet 4 = 16$).

   Both terms are perfect squares, and they are being subtracted. Thus, this binomial meets the conditions of the difference of two squares.

3. Use the pattern $x^2 - y^2 = (x + y)(x - y)$ to factor the given binomial.

   Since the binomial $9x^2 - 16$ meets the conditions of the difference of two squares, it can be factored using the pattern $x^2 - y^2 = (x + y)(x - y)$.

   Notice that the square root of the first term becomes the first term in both factors and the square root of the second term becomes the second term in both factors. In one factor the terms are added, and in the other they are subtracted.

   Substitute $3x$ and 4 (the square roots of $9x^2$ and 16) for x and y in the difference of two squares formula to factor the binomial.

$x^2 - y^2 = (x + y)(x - y)$	Difference of two squares
$(9x^2 - 16) = (x + y)(x - y)$	Substitute $9x^2 - 16$ for $x^2 - y^2$.
$(9x^2 - 16) = [(3x) + (4)][(3x) - (4)]$	Substitute $3x$ for x and 4 for y.

   The factors of $9x^2 - 16$ are $(3x + 4)$ and $(3x - 4)$.

4. Multiply the factors to verify that they result in the original binomial.

   Factors must always equal the original expression when multiplied.

$(3x + 4)(3x - 4)$	Multiply the factors.
$= 3x(3x) + 3x(-4) + 4(3x) + 4(-4)$	Distribute.
$= 9x^2 - 12x + 12x - 16$	Multiply.
$= 9x^2 - 16$	Combine like terms.

   The product of $(3x + 4)(3x - 4)$ equals the original expression, $9x^2 - 16$.

   Therefore, it is confirmed that $9x^2 - 16$ factors to $(3x + 4)(3x - 4)$.

Example 2

Factor $5y^2 - 45$, and then verify your results.

1. Determine any common factors of the given binomial, if common factors exist.

   The prime factors of $5y^2$ are 5, y, and y.

   The prime factors of 45 are 3, 3, and 5.

   5 is the greatest common factor of these two terms and can be factored out:

   $5y^2 - 45$

   $= 5 \bullet y^2 - 5 \bullet 9$

   $= 5(y^2 - 9)$

   When factored by the greatest common factor, $5y^2 - 45 = 5(y^2 - 9)$.

2. Determine if the binomial $y^2 - 9$ meets the conditions of the difference of two squares.

   Binomials that are the difference of two squares are of the form $x^2 - y^2$.

   y^2 is a perfect square because y^2 is the product of two equal factors ($y \bullet y = y^2$).

   9 is also a perfect square because 9 is the product of two equal factors ($3 \bullet 3 = 9$).

   Both terms are perfect squares, and they are being subtracted. Thus, this binomial meets the conditions of the difference of two squares.

3. Use the pattern $x^2 - y^2 = (x + y)(x - y)$ to factor the binomial.

   Since the binomial term of $5(y^2 - 9)$ meets the conditions of the difference of two squares, it can be factored using the pattern $x^2 - y^2 = (x + y)(x - y)$.

   Substitute y and 3 (the square roots of y^2 and 9) for x and y in the difference of two squares formula to factor the binomial.

$x^2 - y^2 = (x + y)(x - y)$	Difference of two squares
$(y^2 - 9) = (x + y)(x - y)$	Substitute $(y^2 - 9)$ for $x^2 - y^2$.
$(y^2 - 9) = [(y) + (3)][(y) - (3)]$	Substitute y for x and 3 for y.
$(y^2 - 9) = (y + 3)(y - 3)$	

   The factors of $5y^2 - 45$ are 5, $(y + 3)$, and $(y - 3)$.

4. Multiply the factors to verify they result in the original binomial.

   Factors must always equal the original expression when multiplied.

$5(y + 3)(y - 3)$	Multiply the factors.
$= 5(y \bullet y + y \bullet -3 + 3 \bullet y + 3 \bullet -3)$	Distribute the two binomials.
$= 5(y^2 - 3y + 3y - 9)$	Multiply.
$= 5(y^2 - 9)$	Combine like terms.
$= 5 \bullet y^2 + 5 \bullet -9$	Distribute the 5.
$= 5y^2 - 45$	Simplify.

   The product of $5(y + 3)(y - 3)$ equals the original expression, $5y^2 - 45$.

   This confirms that $5y^2 - 45$ factors to $5(y + 3)(y - 3)$.

Example 3

The polynomial expression $x^2 + 7x - 8$ represents the area in square feet of the Bingham family's rectangular backyard. Factor this polynomial to find the expressions that represent the length and the width of the backyard, and then verify your results.

1. Determine any common factors of the given trinomial, if common factors exist.

   The prime factors of x^2 are x and x.

   The prime factors of $7x$ are 7 and x.

   The prime factors of –8 are –2, 2, and 2.

   There are no factors that are common to all three terms.

2. Set up two sets of parentheses and write the variable as the first term in each set of parentheses.

   The variable is x, so x will be the first term in each set of parentheses:

   $(x \quad)(x \quad)$

3. List all of the possible sets of factors for the constant term (include both positive and negative numbers).

   The constant term of $x^2 + 7x - 8$ is –8. Possible sets of factors of –8:

   $(2 \bullet -4)$ $(-2 \bullet 4)$ $(1 \bullet -8)$ $(-1 \bullet 8)$

4. Determine which set of factors has a sum that equals the coefficient of the first-degree term.

   The coefficient of the first-degree term of $x^2 + 7x - 8$ is 7.

   Determine which set of factors has a sum of 7:

   - For $(2 \bullet -4)$, $2 + (-4) = -2$, so this is not the correct set of factors.
   - For $(-2 \bullet 4)$, $-2 + 4 = 2$, so this is not the right set of factors.
   - For $(1 \bullet -8)$, $1 + (-8) = -7$, so this is not the right set of factors.
   - For $(-1 \bullet 8)$, $-1 + 8 = 7$. This is the correct set of factors.

   The right set of factors includes –1 and 8.

5. Write the correct factors (including their signs) as the second terms in the two sets of parentheses.

   In step 2, we set up the parentheses and variables for the factors as $(x \quad)(x \quad)$.

   Write –1 and 8 as the second terms in these two parentheses:

   $(x - 1)(x + 8)$

   The factors of $x^2 + 7x - 8$ are $(x - 1)$ and $(x + 8)$.

6. Distribute the factors to verify that the result is the original trinomial.

   Factors must always equal the original expression when multiplied.

$(x - 1)(x + 8)$	Multiply the factors.
$= x(x) + x(8) + (-1)(x) + (-1)(8)$	Distribute.
$= x^2 + 8x - 1x - 8$	Multiply.
$= x^2 + 7x - 8$	Combine like terms.

   The product of $(x - 1)(x + 8)$ equals the original expression, $x^2 + 7x - 8$.

   This confirms that $x^2 + 7x - 8$ factors to $(x - 1)(x + 8)$.

   Therefore, the expressions that represent the length and width of the backyard are $(x - 1)$ feet and $(x + 8)$ feet.

Example 4

Factor $2a^2 - 16a + 32$, and then verify your results.

1. Determine any common factors of the given trinomial, if common factors exist.

   The prime factors of $2a^2$ are 2, a, and a.

   The prime factors of $-16a$ are –2, 2, 2, 2, and a.

   The prime factors of 32 are 2, 2, 2, 2, and 2.

   The greatest common factor of all three terms is 2. Thus, 2 must be factored out:

   $2a^2 - 16a + 32$

   $= 2 \bullet a^2 - 2 \bullet 8a + 2 \bullet 16$

   $= 2(a^2 - 8a + 16)$

   When factored by the greatest common factor, $2a^2 - 16a + 32 = 2(a^2 - 8a + 16)$.

2. Set up two sets of parentheses and write the variable as the first term in each set of parentheses.

   The variable is a, so a will be the first term in each set of parentheses. Remember to include the 2 that was previously factored: $2(a \quad)(a \quad)$.

3. List all of the possible sets of factors for the constant term (include both positive and negative numbers).

   The constant term of the trinomial $a^2 - 8a + 16$ is 16. Possible sets of factors of 16:

$(4 \bullet 4)$	$(2 \bullet 8)$	$(1 \bullet 16)$
$(-4 \bullet -4)$	$(-2 \bullet -8)$	$(-1 \bullet -16)$

4. Determine which set of factors has a sum that equals the coefficient of the first-degree term.

   The coefficient of the first-degree term of $a^2 - 8a + 16$ is –8.

   Determine which set of factors has a sum of –8:

   - For $(4 \bullet 4)$, $4 + 4 = 8$, so this is not the correct set of factors.
   - For $(2 \bullet 8)$, $2 + 8 = 10$, so this is not the right set of factors either.
   - For $(1 \bullet 16)$, $1 + 16 = 17$, so this is not the right set of factors.
   - For $(-4 \bullet -4)$, $-4 + (-4) = -8$. This is the correct set of factors.

   Once the correct set of factors is found, there is no need to continue testing other factors.

   The correct set of factors is –4 and –4.

5. Write the correct factors (including their signs) as the second terms in the two sets of parentheses.

   Previously, we set up the parentheses and variables for the factors as $2(a \quad)(a \quad)$.

   Write –4 and –4 as the second terms in these two parentheses:

   $2(a - 4)(a - 4)$

   This can be written as $2(a - 4)^2$.

   The trinomial $2a^2 - 16a + 32$ factors to $2(a - 4)^2$.

   Notice that since $a^2 - 8a + 16$ is a perfect square trinomial, this could also be factored using the pattern $(ax)^2 \pm 2abx + b^2 = (a \pm b)^2$.

6. Distribute the factors to verify that they result in the original trinomial.

   Factors must always equal the original expression when multiplied.

$2(a-4)^2$	Factors
$= 2(a-4)(a-4)$	Expand the power.
$= 2(a \bullet a + a \bullet -4 + -4 \bullet a + -4 \bullet -4)$	Distribute the binomials.
$= 2(a^2 - 4a - 4a + 16)$	Multiply.
$= 2(a^2 - 8a + 16)$	Combine like terms.
$= 2(a^2) + 2(-8a) + 2(16)$	Distribute the 2.
$= 2a^2 - 16a + 32$	Multiply.

   The product of $2(a-4)^2$ equals the original expression, $2a^2 - 16a + 32$.

   This confirms that $2a^2 - 16a + 32$ factors to $2(a-4)^2$.

Example 5

Factor $x^2 - 18x + 81$, and then verify your results.

1. Determine any common factors of the given trinomial, if common factors exist.

   The prime factors of x^2 are $x \bullet x$.

   The prime factors of $-18x$ are $-2 \bullet 3 \bullet 3 \bullet x$.

   The prime factors of 81 are $3 \bullet 3 \bullet 3 \bullet 3$.

   There are no factors that are common to all three terms.

2. Determine if the trinomial is a perfect square trinomial.

   Trinomials that are the difference of two squares are of the form $(ax)^2 \pm 2abx + b^2$.

   In other words, the first and last terms are both perfect squares and the middle term is equal to twice the product of the square roots of the first and last terms.

   For $x^2 - 18x + 81$, both the first term (x^2) and last term (81) are perfect squares since they are both products of two equal factors.

   The square root of x^2 is x and the square root of 81 is 9. Thus, the middle term is twice the product of the square roots of the first and last terms ($2 \bullet x \bullet 9 = 18x$).

3. Use the pattern for the perfect square trinomial to factor this expression.

   Perfect square trinomials can be factored using the regular process for factoring trinomials. Or, they can be factored using the pattern $(ax)^2 \pm 2abx + b^2 = (a \pm b)^2$.

   For the trinomial $x^2 - 18x + 81$, $a = 1$ (the square root of the coefficient of x^2) and $b = 9$ (the square root of the constant term 81).

   Since the middle term, $-18x$, is negative, the sign within the factored form will also be negative.

   $x^2 - 18x + 81 = (x - 9)^2$

4. Distribute the factors to verify that they result in the original trinomial.

   Factors must always equal the original expression when multiplied.

$(x - 9)^2$	Factors
$= (x - 9)(x - 9)$	Expand the power.
$= (x \bullet x + x \bullet -9 + -9 \bullet x + -9 \bullet -9)$	Distribute the binomials.
$= x^2 - 9x - 9x + 81$	Multiply.
$= x^2 - 18x + 81$	Combine like terms.

   The product of $(x - 9)^2$ equals the original expression, $x^2 - 18x + 81$.

   This confirms that $x^2 - 18x + 81$ factors to $(x - 9)^2$.