Guided Practice 5.5

Example 1

Factor $5y^2 - 6y + 10y - 12$ by grouping, and then verify your results.

1. Group the terms of the given polynomial according to their common factors, if common factors exist.

   Since $5y^2$ and $10y$ have a common factor of $5y$, group these two terms. Likewise, group $-6y$ and -12 because they have a common factor of -6:

   $5y^2 - 6y + 10y - 12 = (5y^2 + 10y) + (-6y - 12)$

2. Factor out common factors from each set of grouped terms, and rewrite the expression from the previous step.

   Rewrite $5y^2 + 10y$ using the common factor of $5y$.

   $5y^2 + 10y$

   $= 5y \bullet y + 5y \bullet 2$

   $= 5y(y + 2)$

   Rewrite $-6y - 12$ using the common factor of -6.

   $-6y - 12$

   $= -6 \bullet y + (-6) \bullet 2$

   $= -6(y + 2)$

   Therefore, $(5y^2 + 10y) + (-6y - 12) = 5y(y + 2) - 6(y + 2)$.

3. Use the Distributive Property to rewrite the expression as two factors.

   Since both groups in the expression $5y(y + 2) - 6(y + 2)$ contain the binomial factor $(y + 2)$, this expression can be rewritten using the Distributive Property.

   The Distributive Property states that $ac + bc = (a + b)c$.

   Thus, $5y(y + 2) - 6(y + 2) = (5y - 6)(y + 2)$.

4. Distribute the result of the previous step to confirm that the original polynomial was factored correctly.

   When multiplied, $(5y - 6)(y + 2)$ should equal $5y^2 - 6y + 10y - 12$.

$(5y - 6)(y + 2)$	Factors
$= 5y(y) + 5y(2) + (-6)(y) + (-6)(2)$	Distribute.
$= 5y^2 + 10y - 6y - 12$	Multiply.
$= 5y^2 - 6y + 10y - 12$	Rearrange the terms to match the original expression.

   The product of $(5y - 6)(y + 2)$ equals the original expression, $5y^2 - 6y + 10y - 12$.

   This confirms that $5y^2 - 6y + 10y - 12$ factors to $(5y - 6)(y + 2)$.

Example 2

Factor $6x^2 - 7x - 5$, and then verify your results.

1. Determine any common factors of the given trinomial, if common factors exist.

   The prime factors of $6x^2$ are 2, 3, x, and x.

   The prime factors of $-7x$ are –7 and x.

   5 is already a prime number.

   There are no factors that are common to all three terms.

2. Multiply a, the coefficient of the second-degree term, by c, the constant.

   Consider the general form $ax^2 + bx + c$.

   Within the trinomial $6x^2 - 7x - 5$, the coefficient of the second-degree term, a, is 6 and the constant, c, is –5.

   Therefore, $a \bullet c$ is equal to $6 \bullet -5$, or –30.

3. List all of the possible factors for the product of a and c.

   The product of a and c is –30.

   Determine the possible sets of factors for –30, including negative factors:

$(1 \bullet -30)$	$(-1 \bullet 30)$	$(2 \bullet -15)$	$(-2 \bullet 15)$
$(3 \bullet -10)$	$(-3 \bullet 10)$	$(5 \bullet -6)$	$(-5 \bullet 6)$

4. Determine which set of factors has a sum that equals b, the coefficient of the first-degree term.

   In the trinomial $6x^2 - 7x - 5$, the coefficient of b, the first-degree term, is –7.

   Determine which set of factors has a sum of –7.

   The set of factors whose sum is –7 is 3 and –10 $(3 + (-10) = -7)$.

   The correct set of factors is 3 and –10.

5. Rewrite bx, the first-degree term, as the sum of two terms.

   Within the trinomial $6x^2 - 7x - 5$, rewrite the first-degree term $-7x$ as $3x - 10x$:

   $6x^2 + 3x - 10x - 5$

6. Group the terms of the new polynomial according to their common factors, if common factors exist.

   Group $6x^2 + 3x - 10x - 5$ so that terms with common factors are together.

   $6x^2$ and $3x$ have a common factor of $3x$, and $-10x$ and -5 have a common factor of -5. Group the terms accordingly:

   $6x^2 + 3x - 10x - 5 = (6x^2 + 3x) + (-10x - 5)$

7. Factor out common factors from each set of grouped terms, and rewrite the expression from the previous step.

   Rewrite $6x^2 + 3x$ using the common factor of $3x$.

   $6x^2 + 3x = 3x \bullet 2x + 3x \bullet 1 = 3x(2x + 1)$

   Rewrite $-10x - 5$ using the common factor of -5.

   $-10x - 5 = -5 \bullet 2x + -5 \bullet 1 = -5(2x + 1)$

   Thus, $(6x^2 + 3x) + (-10x - 5)$ can be rewritten as $3x(2x + 1) - 5(2x + 1)$.

8. Use the Distributive Property to rewrite the expression as two factors.

   Recall that the Distributive Property states that $ac + bc = (a + b)c$.

   $3x(2x + 1) - 5(2x + 1) = (3x - 5)(2x + 1)$

9. Distribute the factors to verify that they result in the original trinomial.

   Factors must always equal the original expression when multiplied.

$(3x - 5)(2x + 1)$	Factors
$= 3x(2x) + 3x(1) + (-5)(2x) + (-5)(1)$	Distribute.
$= 6x^2 + 3x - 10x - 5$	Multiply.
$= 6x^2 - 7x - 5$	Combine like terms.

   The product of $(3x - 5)(2x + 1)$ equals the original expression, $6x^2 - 7x - 5$.

   This confirms that $6x^2 - 7x - 5$ factors to $(3x - 5)(2x + 1)$.

Example 3

Factor $36x^2 - 54x + 8$, and then verify your results.

1. Determine any common factors of the given trinomial, if common factors exist.

   The prime factors of $36x^2$ are 2, 2, 3, 3x, and x.

   The prime factors of $-54x$ are -1, 2, 3, 3, 3, and x.

   The prime factors of 8 are 2, 2, and 2.

   2 is the greatest common factor of all three terms and can be factored out:

   $36x^2 - 54x + 8$

   $= 2 \bullet 18x^2 + 2 \bullet -27x + 2 \bullet 4$

   $= 2(18x^2 - 27x + 4)$

   When factored by the greatest common factor, $36x^2 - 54x + 8 = 2(18x^2 - 27x + 4)$.

   Next, factor the remaining trinomial.

2. Multiply a, the coefficient of the second-degree term, by c, the constant.

   Consider the general form $ax^2 + bx + c$.

   Within the trinomial $18x^2 - 27x + 4$, the coefficient of the second-degree term, a, is 18 and the constant, c, is 4.

   Therefore, $a \bullet c$ is equal to $18 \bullet 4$, or 72.

3. List all of the possible factors for the product of a and c.

   Determine the possible sets of factors for 72, including negative factors:

$(1 \bullet 72)$	$(2 \bullet 36)$	$(3 \bullet 24)$	$(4 \bullet 18)$
$(6 \bullet 12)$	$(8 \bullet 9)$	$(-1 \bullet -72)$	$(-2 \bullet -36)$
$(-3 \bullet -24)$	$(-4 \bullet -18)$	$(-6 \bullet -12)$	$(-8 \bullet -9)$

4. Determine which set of factors has a sum that equals b, the coefficient of the first-degree term.

   In the trinomial $18x^2 - 27x + 4$, the coefficient of the first-degree term, b, is -27.

   Determine which set of factors has a sum of -27.

   The set of factors whose sum is -27 is -3 and -24 $(-3 + (-24) = -27)$.

   The correct set of factors is -3 and -24.

5. Rewrite bx, the first-degree term, as the sum of two terms.

   Within the trinomial $18x^2 - 27x + 4$, rewrite the first-degree term $-27x$ as $-3x - 24x$:

   $18x^2 - 3x - 24x + 4$

6. Group the terms of the new polynomial according to their common factors, if common factors exist.

   Group $18x^2 - 3x - 24x + 4$ so that terms with common factors are grouped together.

   $6x^2$ and $-3x$ have a common factor of $3x$, and $-24x$ and 4 have a common factor of 4.

   Group the terms accordingly:

   $18x^2 - 3x - 24x + 4 = (18x^2 - 3x) + (-24x + 4)$

7. Factor out common factors from each set of grouped terms, and rewrite the expression from the previous step.

   Rewrite $18x^2 - 3x$ using the common factor of $3x$.

   $18x^2 - 3x = 3x \bullet 6x - 3x \bullet 1 = 3x(6x - 1)$

   Rewrite $-24x + 4$ using the common factor of 4.

   $-24x + 4 = 4(-6x) + 4 \bullet 1 = 4(-6x + 1)$

   Therefore, $(18x^2 - 3x) + (-24x + 4)$ can be rewritten as $3x(6x - 1) + 4(-6x + 1)$.

8. Factor the expressions further to create matching binomial factors.

The two binomials do not match, but if –1 is factored out of the second binomial, then the binomials will match.

Factor –1 out of $(-6x + 1)$:

$4(-6x + 1) = 4(-1 \bullet 6x + -1 \bullet -1) = 4 \bullet -1(6x - 1) = -4(6x - 1)$

The expression is now $3x(6x - 1) - 4(6x - 1)$.

9. Use the Distributive Property to rewrite the expression as two factors, and then write the result along with the GCF determined in step 1.

Recall that the Distributive Property states that $ac + bc = (a + b)c$.

$3x(6x - 1) - 4(6x - 1) = (3x - 4)(6x - 1)$

Write the factored form of $2(18x^2 - 27x + 4)$, remembering to include the GCF, 2, that was previously factored out.

The expression $2(18x^2 - 27x + 4)$ factors to $2(3x - 4)(6x - 1)$.

10. Distribute the factors to verify that they result in the original trinomial.

Factors must always equal the original expression when multiplied.

$2(3x - 4)(6x - 1)$	Factors
$= 2(3x \bullet 6x + 3x \bullet -1 + -4 \bullet 6x + -4 \bullet -1)$	Distribute the binomials.
$= 2(18x^2 - 3x - 24x + 4)$	Multiply.
$= 2(18x^2 - 27x + 4)$	Combine like terms.
$= 2 \bullet 18x^2 + 2 \bullet -27x + 2 \bullet 4$	Distribute the 2.
$= 36x^2 - 54x + 8$	Multiply.

The product of $2(3x - 4)(6x - 1)$ equals the original expression, $36x^2 - 54x + 8$.

This confirms that $36x^2 - 54x + 8$ factors to $2(3x - 4)(6x - 1)$.

Example 4

The polynomial expression $10a^2 + 87a - 27$ represents the area in square yards of a new rectangular playground at the town park. Factor the polynomial to determine the expressions that represent the length and the width of the playground. Verify your results.

1. Determine any common factors of the given trinomial, if common factors exist.

   The prime factors of $10a^2$ are 2, 5, a, and a.

   The prime factors of $87a$ are 3, 29, and a.

   The prime factors of 27 are 3, 3, and 3.

   There are no factors that are common to all three terms.

2. Multiply a, the coefficient of the second-degree term, by c, the constant.

   Consider the general form $ax^2 + bx + c$.

   Within the trinomial $10a^2 + 87a - 27$, a, the coefficient of the second-degree term, is 10. The constant, c, is –27.

   Therefore, $a \bullet c$ is equal to $10 \bullet -27$, or –270.

3. List all of the possible factors for the product of a and c.

   The possible sets of factors for –270, including negative factors, are:

$(1 \bullet -270)$	$(-1 \bullet 270)$	$(2 \bullet -135)$	$(-2 \bullet 135)$
$(3 \bullet -90)$	$(-3 \bullet 90)$	$(5 \bullet -54)$	$(-5 \bullet 54)$
$(6 \bullet -45)$	$(-6 \bullet 45)$	$(9 \bullet -30)$	$(-9 \bullet 30)$
$(10 \bullet -27)$	$(-10 \bullet 27)$	$(15 \bullet -18)$	$(-15 \bullet 18)$

4. Determine which set of factors has a sum that equals b, the coefficient of the first-degree term.

   In the trinomial $10a^2 + 87a - 27$, the coefficient of b, the first-degree term, is 87.

   Determine which set of factors has a sum of 87.

   The set of factors whose sum is 87 is –3 and 90 ($-3 + 90 = 87$).

   The correct set of factors is –3 and 90.

5. Rewrite bx, the first-degree term, as the sum of two terms.

   Within the trinomial $10a^2 + 87a - 27$, rewrite the first-degree term, $87a$, as $-3a + 90a$: $10a^2 - 3a + 90a - 27$.

6. Group the terms of the new polynomial according to their common factors, if common factors exist.

   Group $10a^2 - 3a + 90a - 27$ so that terms with common factors are grouped together.

   $10a^2$ and $90a$ have a common factor of $10a$, and $-3a$ and -27 have a common factor of -3.

   Group the terms accordingly:

   $$10a^2 - 3a + 90a - 27 = (10a^2 + 90a) + (-3a - 27)$$

7. Factor out common factors from each set of grouped terms, and rewrite the expression from the previous step.

   Rewrite $10a^2 + 90a$ using the common factor of $10a$.

   $10a^2 + 90a = 10a \bullet a + 10a \bullet 9 = 10a(a + 9)$

   Rewrite $-3a - 27$ using the common factor of -3.

   $-3a - 27 = -3 \bullet a + -3 \bullet 9 = -3(a + 9)$

   Thus, $(10a^2 + 90a) + (-3a - 27)$ can be rewritten as $10a(a + 9) - 3(a + 9)$.

8. Use the Distributive Property to rewrite the expression as two factors.

   Recall that the Distributive Property states that $ac + bc = (a + b)c$.

   $10a(a + 9) - 3(a + 9) = (10a - 3)(a + 9)$

9. Distribute the factors to verify that they result in the original trinomial.

   Factors must always equal the original expression when multiplied.

$(10a - 3)(a + 9)$	Factors
$= 10a(a) + 10a(9) + (-3)(a) + (-3)(9)$	Distribute.
$= 10a^2 + 90a - 3a - 27$	Multiply.
$= 10a^2 + 87a - 27$	Combine like terms.

   The product of $(10a - 3)(a + 9)$ equals the original expression, $10a^2 + 87a - 27$.

   This confirms that $10a^2 + 87a - 27$ factors to $(10a - 3)(a + 9)$.

   The expressions that represent the length and width of the playground are $(10a - 3)$ yards and $(a + 9)$ yards.