Guided Practice 5.6

Example 1

Solve $x^2 + 3x = 0$ for x.

1. Write the equation in standard form, $ax^2 + bx + c = 0$, and determine values for a, b, and c.

   In this case, the equation is already in standard form because the polynomial on the left, $x^2 + 3x$, is set equal to 0.

   For $x^2 + 3x = 0$, there is an understood coefficient of 1 for the x^2 term, so $a = 1$ and $b = 3$. There is no constant, so $c = 0$.

2. Factor the polynomial and rewrite the equation.

   First, determine if $x^2 + 3x$ has any common factors.

   Both terms have a factor of x; therefore,
   $x^2 + 3x = (x \bullet x) + (x \bullet 3) = x(x + 3)$.

   The polynomial cannot be factored any further.

   The factors of $x^2 + 3x$ are x and $(x + 3)$.

   The equation $x^2 + 3x = 0$ can be rewritten as $x(x + 3) = 0$.

3. Set each factor equal to 0 and solve for x.

   Once the polynomial has been factored, set each factor equal to 0 and then solve for x.

   $x = 0$ $\qquad$ $x + 3 = 0$

   $\qquad\qquad$ $x = -3$

   The solutions to the equation $x^2 + 3x = 0$ are $x = 0$ and $x = -3$.

4. Check the solutions by substituting them into the original equation.

   Check that a solution is correct by substituting it back into the original equation and solving the equation to ensure that it results in a true statement.

   Check $x = 0$:

$x^2 + 3x = 0$	Original equation
$(0)^2 + 3(0) = 0$	Substitute 0 for x.
$0 + 0 = 0$	Evaluate the power and multiply.
$0 = 0$	Add.

   Because $x = 0$ makes the equation true, it is a solution to the equation.

   Check $x = -3$:

$x^2 + 3x = 0$	Original equation
$(-3)^2 + 3(-3) = 0$	Substitute –3 for x.
$9 + (-9) = 0$	Evaluate the power and multiply.
$0 = 0$	Add.

   Because $x = -3$ makes the equation true, it is also a solution.

   The solutions to the equation $x^2 + 3x = 0$ are $x = 0$ and $x = -3$.

Example 2

Solve $4x^2 - 26x = 14$ for x.

1. Write the equation in standard form, $ax^2 + bx + c = 0$, and determine values for a, b, and c.

   The right side of the equation must equal 0. To get 0 on the right side, subtract 14 from both sides: $4x^2 - 26x - 14 = 0$.

   The equation is now in standard form because the polynomial on the left, $4x^2 - 26x - 14$, is set equal to 0.

   For $4x^2 - 26x - 14 = 0$, $a = 4$, $b = -26$, and $c = -14$.

2. Factor the polynomial and rewrite the equation.

   First, determine if $4x^2 - 26x - 14$ has any common factors.

   All three terms have a factor of 2; therefore,
   $4x^2 - 26x - 14 = 2 \bullet 2x^2 - 2 \bullet 13x - 2 \bullet 7 = 2(2x^2 - 13x - 7)$.

   Next, factor the trinomial $2x^2 - 13x - 7$.

   The product of 2 (the coefficient of the second-degree term) and –7 (the constant) is –14. Find the factors of –14 that add up to –13, the coefficient of the first-degree term.

   –14 and 1 are factors of –14 that have a sum of –13.

   Rewrite $-13x$ as $-14x + 1x$:

   $2x^2 - 13x - 7 = 2x^2 - 14x + 1x - 7$

   Factor $2x^2 - 14x + 1x - 7$ by grouping.

$2x^2 - 14x + 1x - 7$	Expression
$= (2x^2 - 14x) + (x - 7)$	Group $2x^2$ and $-14x$ because they have a common factor of $2x$.
$= 2x(x - 7) + (x - 7)$	Factor out $2x$ from $2x^2 - 14x$.
$= (2x + 1)(x - 7)$	Use the Distributive Property to rewrite the expression as two factors.

   The factors of $4x^2 - 26x - 14$ are 2, $(2x + 1)$, and $(x - 7)$.

   The equation $4x^2 - 26x - 14 = 0$ can be rewritten as $2(2x + 1)(x - 7) = 0$.

3. Set each factor equal to 0 and solve for x.

Once the polynomial has been factored, set each factor equal to 0.

The factor 2 is a constant and can be disregarded because 2 cannot equal 0. Furthermore, if both sides of the equation were divided by 2, the 2 would disappear since 0 divided by 2 is 0.

Only the factors that contain a variable can be set equal to 0 and solved. Set both of these factors equal to 0 and solve:

$2x + 1 = 0$	$x - 7 = 0$
$2x = -1$	$x = 7$
$x = -\frac{1}{2}$	

The solutions to the equation $4x^2 - 26x = 14$ are $x = -\frac{1}{2}$ and $x = 7$.

4. Check the solutions by substituting them into the original equation.

Check that a solution is correct by substituting it back into the original equation and solving the equation to ensure that it results in a true statement.

Check $x = -\frac{1}{2}$:

$4x^2 - 26x = 14$	Original equation
$4\left(-\frac{1}{2}\right)^2 - 26\left(-\frac{1}{2}\right) = 14$	Substitute $-\frac{1}{2}$ for x.
$4\left(\frac{1}{4}\right) - 26\left(-\frac{1}{2}\right) = 14$	Evaluate the power.
$1 + 13 = 14$	Multiply.
$14 = 14$	Add.

Because $x = -\frac{1}{2}$ makes the equation true, it is a solution to the equation.

(continued)

Check $x = 7$:

$4x^2 - 26x = 14$	Original equation
$4(7)^2 - 26(7) = 14$	Substitute 7 for x.
$4(49) - 26(7) = 14$	Evaluate the power.
$196 - 182 = 14$	Multiply.
$14 = 14$	Subtract.

Because $x = 7$ makes the equation true, it is also a solution.

The solutions to the equation $4x^2 - 26x = 14$ are $x = -\frac{1}{2}$ and $x = 7$.

Example 3

The area of the Baker family's rectangular driveway is 324 square feet. The length is 3 feet larger than twice the width. What are the length and width of the driveway?

1. Write expressions that represent the length and width of the driveway.

   Let x represent the width of the driveway.

   The length is 3 feet larger than twice the width. Thus, the length must equal $2x + 3$.

   x represents the width of the driveway and $2x + 3$ represents the length.

2. Write an equation for the area of the driveway.

   Use the fact that the area of a rectangle equals the length times the width to write an equation.

$A = lw$	Formula for the area of a rectangle
$(324) = (2x + 3)(x)$	Substitute 324 for the area, $(2x + 3)$ for the length, and x for the width.

   The equation representing the area of the driveway is $324 = (2x + 3)(x)$.

3. Write the equation in standard form, $ax^2 + bx + c = 0$.

   Rearrange the equation so that the polynomial is in standard form, set equal to 0.

$324 = (2x + 3)(x)$	Original equation
$(2x + 3)(x) = 324$	Apply the Symmetric Property of Equality to switch the left and right sides.
$2x^2 + 3x = 324$	Distribute the x.
$2x^2 + 3x - 324 = 0$	Subtract 324 from both sides.

   The equation $2x^2 + 3x - 324 = 0$ is now in standard form and is ready to be factored.

4. Factor the polynomial and rewrite the equation.

   First, determine if $2x^2 + 3x - 324$ has any common factors.

   There are no factors that are common to all three terms.

   Next, factor the trinomial $2x^2 + 3x - 324$.

   The product of 2 (the coefficient of the second-degree term) and –324 (the constant) is –648. Find the factors of –648 that add up to 3, the coefficient of the first-degree term.

   27 and –24 are factors of –648 that have a sum of 3.

   Rewrite $3x$ as $27x - 24x$:

   $$2x^2 + 3x - 324 = 2x^2 + 27x - 24x - 324$$

   Factor $2x^2 + 27x - 24x - 324$ by grouping. Note that $2x^2$ and $-24x$ have a common factor of $2x$, and $27x$ and –324 have a common factor of 27.

$2x^2 + 27x - 24x - 324$	Expression
$= (2x^2 - 24x) + (27x - 324)$	Group according to common factors.
$= 2x(x - 12) + 27(x - 12)$	Factor out $2x$ from $2x^2 - 24x$ and 27 from $27x - 324$.
$= (2x + 27)(x - 12)$	Use the Distributive Property to rewrite the expression as two factors.

   The factors of $2x^2 + 3x - 324$ are $(2x + 27)$ and $(x - 12)$.

   The equation $2x^2 + 3x - 324 = 0$ can be rewritten as $(2x + 27)(x - 12) = 0$.

5. Set each factor equal to 0 and solve for x.

   Once the polynomial has been factored, set each factor equal to 0.

   The first factor is $(2x + 27)$. Set this equal to 0: $2x + 27 = 0$

   The second factor is $(x – 12)$. Set this equal to 0: $x – 12 = 0$

   Next, solve each equation for x.

$2x + 27 = 0$	$x - 12 = 0$
$2x = -27$	$x = 1$
$x = -\dfrac{27}{2}$	

   The solutions to the equation are $x = -\dfrac{27}{2}$ and $x = 12$.

6. Determine which solution for x makes sense for the situation.

   Since x represents the width of the driveway, x must be positive (the width of the driveway cannot be negative). Thus, $x = -\dfrac{27}{2}$ does not make sense as a solution, so it can be disregarded. Therefore, $x = 12$ is the only solution to the equation given the context of the scenario.

7. Find the length and the width of the driveway.

   Since x represents the width of the driveway and $x = 12$, the width of the driveway is 12 feet.

   Since $2x + 3$ represents the length of the driveway, substitute 12 for x and solve to find the length:

$2x + 3$	Length of the driveway
$= 2(12) + 3$	Substitute 12 for x.
$= 24 + 3$	Multiply.
$= 27$	Add.

   The length of the driveway is 27 feet.

   The driveway is 27 feet long and 12 feet wide.