Guided Practice 5.7

Example 1

Find the y-intercept and vertex of the function $f(x) = -2x^2 + 4x + 3$. Determine whether the vertex is a minimum or maximum point on the graph.

1. Determine the y-intercept.

   The function $f(x) = -2x^2 + 4x + 3$ is written in standard form, $f(x) = ax^2 + bx + c$.

   When $x = 0$, the y-intercept equals c, which is 3.

   Verify that 3 is the y-intercept.

$f(x) = -2x^2 + 4x + 3$	Original equation
$f(0) = -2(0)^2 + 4(0) + 3$	Substitute 0 for x.
$f(0) = 3$	Simplify.

   The y-intercept is 3.

2. Determine the vertex of the function.

   $f(x) = -2x^2 + 4x + 3$ is in standard form; therefore, $a = -2$ and $b = 4$.

$x = \frac{-b}{2a}$	The x-coordinate of the vertex
$x = \frac{-4}{2(-2)}$	Substitute –2 for a and 4 for b.
$x = 1$	Simplify.

   The vertex has an x-coordinate of 1.

(*continued*)

Since h is the x-coordinate of the vertex, or the input value of the vertex, we can find the output value, k, by evaluating the function for $x = 1$.

$f(x) = -2x^2 + 4x + 3$	Original equation
$f(1) = -2(1)^2 + 4(1) + 3$	Substitute 1 for x.
$f(1) = 5$	Simplify.

The y-coordinate of the vertex is 5.

The vertex is the point (1, 5).

3. Use the value of a to determine if the vertex is a maximum or a minimum value.

   Given that $f(x) = -2x^2 + 4x + 3$ is written in standard form, $a = -2$.

   Since $a < 0$, the parabola opens down.

   As such, the vertex (1, 5) is a maximum point.

Example 2

$h(x) = 2x^2 - 11x + 5$ is a quadratic function. Determine the direction in which the function opens, the coordinates of the vertex, the axis of symmetry, the x-intercept(s), if any, and the y-intercept. Use this information to sketch the graph.

1. Determine whether the graph opens up or down.

   $h(x) = 2x^2 - 11x + 5$ is in standard form; therefore, $a = 2$.

   Since $a > 0$, the parabola opens up.

2. Find the vertex and the equation of the axis of symmetry.

$h(x) = 2x^2 - 11x + 5$ is in standard form; therefore, $a = 2$ and $b = -11$.

$x=\frac{-b}{2a}$	The x-coordinate of the vertex
$x=\frac{-(-11)}{2(2)}$	Substitute 2 for a and –11 for b.
$x = 2.75$	Simplify.

The vertex has an x-coordinate of 2.75.

Since the input value is 2.75, find the output value by evaluating the function for $x = 2.75$.

$h(x) = 2x^2 - 11x + 5$	Original equation
$h(2.75) = 2(2.75)^2 - 11(2.75) + 5$	Substitute 2.75 for x.
$h(2.75) = -10.125$	Simplify.

The y-coordinate of the vertex is –10.125.

The vertex is the point (2.75, –10.125).

Since the axis of symmetry is the vertical line through the vertex, the equation of the axis of symmetry is $x = 2.75$.

3. Find the y-intercept.

$h(x) = 2x^2 - 11x + 5$ is in standard form, so the y-intercept is the constant term c, which is 5.

The y-intercept is 5.

4. Find the x-intercepts, if any exist.

The x-intercepts occur when $y = 0$.

Substitute 0 for the output, $h(x)$, and solve.

This equation is factorable, but if we cannot easily identify the factors, the quadratic formula always works.

Note both methods.

Solved by factoring:

$h(x) = 2x^2 - 11x + 5$

$0 = 2x^2 - 11x + 5$

$0 = (2x - 1)(x - 5)$

$0 = 2x - 1$ or $0 = x - 5$

$x = 0.5$ or $x = 5$

Solved using the quadratic formula:

$h(x) = 2x^2 - 11x + 5$

$0 = 2x^2 - 11x + 5$

$a = 2$, $b = -11$, and $c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{11 \pm \sqrt{81}}{4}$$

$$x = \frac{11 \pm 9}{4}$$

$$x = \frac{11 - 9}{4} \text{ or } x = \frac{11 + 9}{4}$$

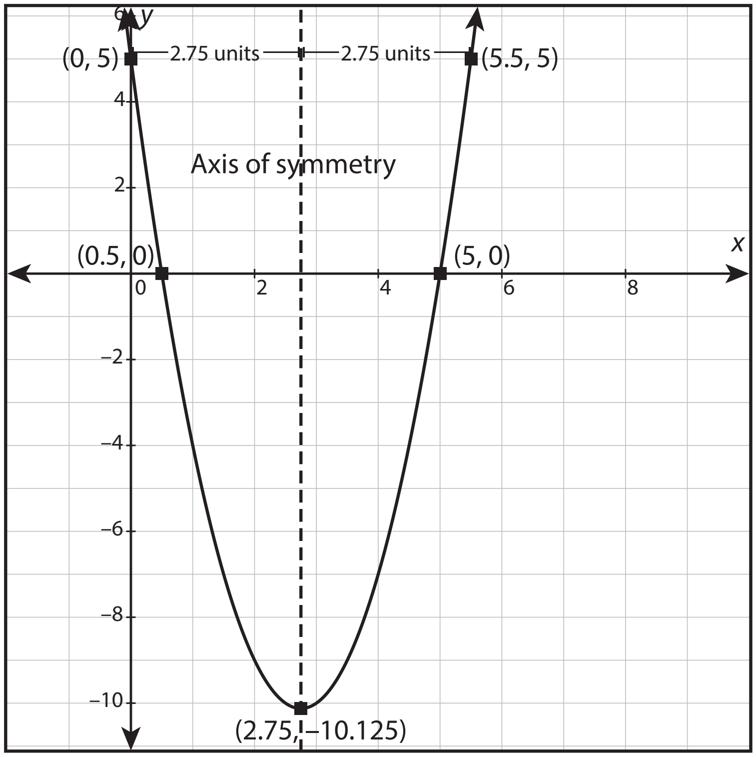
$$x = \frac{2}{4} \text{ or } x = \frac{20}{4}$$

$x = 0.5$ or $x = 5$

The x-intercepts are 0.5 and 5.

5. Plot the points from steps 2–4 and their symmetric points over the axis of symmetry.

   Connect the points with a smooth curve.



Example 3

$R(x) = 2x^2 + 8x + 8$ is a quadratic function. Determine the direction in which the function opens, the vertex, the axis of symmetry, the x-intercept(s), if any, and the y-intercept. Use this information to sketch the graph.

1. Determine whether the graph opens up or down.

   $R(x) = 2x^2 + 8x + 8$ is written in standard form; therefore, $a = 2$.

   Since $a > 0$, the parabola opens up.

2. Find the vertex and the equation of the axis of symmetry.

$R(x) = 2x^2 + 8x + 8$ is in standard form; therefore, $a = 2$ and $b = 8$.

$x = \dfrac{-b}{2a}$	The x-coordinate of the vertex
$x = \dfrac{-(8)}{2(2)}$	Substitute 2 for a and 8 for b.
$x = -2$	Simplify.

The vertex has an x-coordinate of –2.

Since the input value is –2, find the output value by evaluating the function for $x = -2$.

$R(x) = 2x^2 + 8x + 8$	Original equation
$R(-2) = 2(-2)^2 + 8(-2) + 8$	Substitute –2 for x.
$R(-2) = 0$	Simplify.

The y-coordinate of the vertex is 0.

The vertex is the point (–2, 0).

Since the axis of symmetry is the vertical line through the vertex, the equation of the axis of symmetry is $x = -2$.

3. Find the y-intercept.

The function, $R(x) = 2x^2 + 8x + 8$, is in standard form, so the y-intercept is the constant term c, which is 8.

The y-intercept is 8.

4. Find the x-intercepts, if any exist.

   The x-intercepts occur when $y = 0$.

   Substitute 0 for the output, $R(x)$, and solve using the quadratic formula or by factoring.

   Solved by factoring:

   $R(x) = 2x^2 + 8x + 8$

   $0 = 2x^2 + 8x + 8$

   $0 = 2(x^2 + 4x + 4)$

   $0 = 2(x + 2)^2$

   $0 = (x + 2)$

   $x = -2$

   Solved using the quadratic formula:

   $R(x) = 2x^2 + 8x + 8$

   $0 = 2x^2 + 8x + 8$

   $a = 2$, $b = 8$, and $c = 8$

   $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

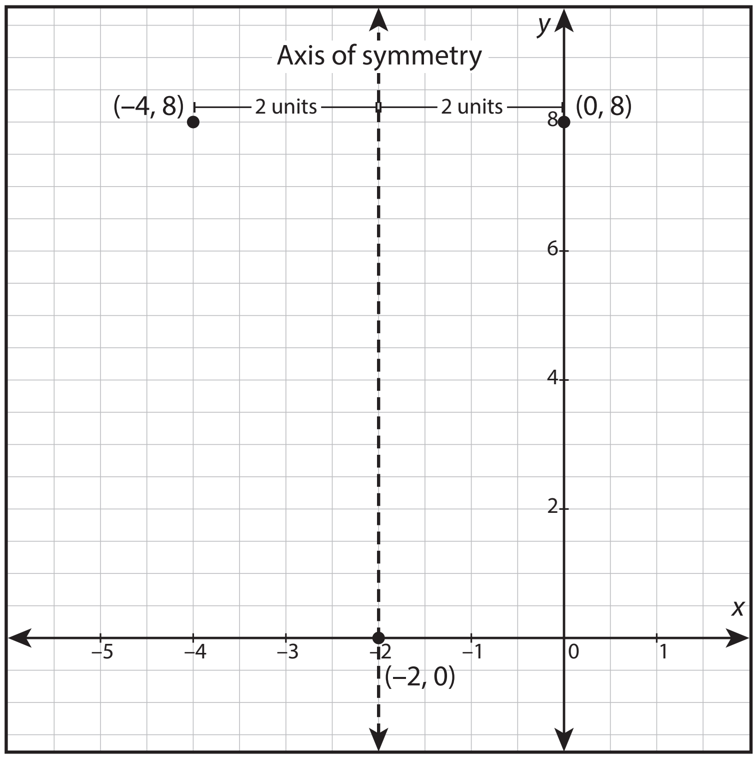
   $$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(2)(8)}}{2(2)}$$

   $$x = \frac{-8 \pm 0}{4}$$

   $x = -2$

   The x-intercept is –2. This is a special case where the vertex is on the x-axis, so there is only one x-intercept.

5. Plot the points from steps 2–4 and their symmetric points over the axis of symmetry.



For a more accurate graph, determine an additional pair of symmetric points.

Choose any *x*-coordinate on the left or right of the axis of symmetry.

Evaluate the function for the chosen value of *x* to determine the output value.

Let's choose $x = -1$.

$R(x) = 2x^2 + 8x + 8$	Original equation
$R(-1) = 2(-1)^2 + 8(-1) + 8$	Substitute –1 for *x*.
$R(-1) = 2$	Simplify.

(–1, 2) is an additional point that lies on the parabola.

Plot (–1, 2) on the same graph.

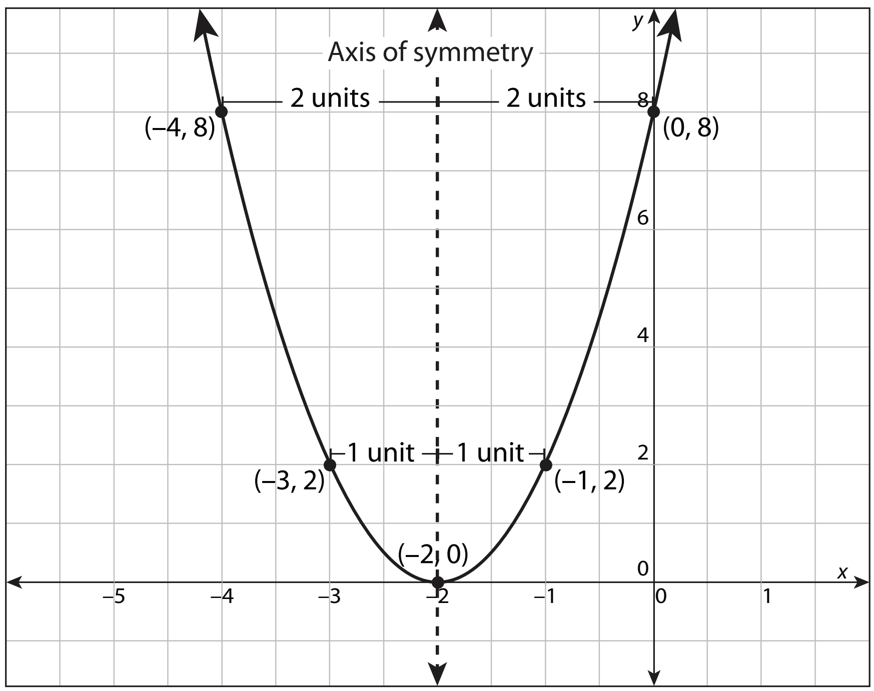
(–1, 2) is 1 unit from the axis of symmetry.

Locate the reflection of (–1, 2) about the axis of symmetry.

(*continued*)

(–3, 2) is also 1 unit from the axis of symmetry and is symmetrical to the original point (–1, 2) with respect to the axis of symmetry.

You can verify that (0, 8) and (–4, 8) are the same distance from the axis of symmetry and are also symmetrical with respect to the axis of symmetry by referring to the graph.



Example 4

$g(x) = -x^2 + 8x - 17$ is a quadratic function. Determine the direction in which the function opens, the vertex, the equation of the axis of symmetry, the x-intercept(s), if any, and the y-intercept. Use this information to sketch the graph.

1. Determine whether the graph opens up or down.

   $g(x) = -x^2 + 8x - 17$ is written in standard form; therefore, $a = -1$.

   Since $a < 0$, the parabola opens down.

2. Find the vertex and the equation of the axis of symmetry.

$g(x) = -x^2 + 8x - 17$ is written in standard form; therefore, $a = -1$ and $b = 8$.

$x = \frac{-b}{2a}$	The x-coordinate of the vertex
$x = \frac{-(8)}{2(-1)}$	Substitute –1 for a and 8 for b.
$x = 4$	Simplify.

The vertex has an x-coordinate of 4.

Since the input value is 4, find the output value by evaluating the function for $x = 4$.

$g(x) = -x^2 + 8x - 17$	Original equation
$g(4) = -(4)^2 + 8(4) - 17$	Substitute 4 for x.
$g(4) = -1$	Simplify.

The y-coordinate of the vertex is –1.

The vertex is the point (4, –1).

Since the axis of symmetry is the vertical line through the vertex, the equation of the axis of symmetry is $x = 4$.

3. Find the y-intercept.

The function $g(x) = -x^2 + 8x - 17$ is in standard form, so the y-intercept is the constant term c, which equals –17.

The y-intercept is –17.

4. Find the x-intercepts, if any exist.

   The x-intercepts occur when $y = 0$.

   Substitute 0 for the output, $g(x)$, and solve using the quadratic formula since the function is not factorable over the rational numbers.

$g(x) = -x^2 + 8x - 17$	Original equation
$-x^2 + 8x - 17 = 0$	Set the equation equal to 0.

   Determine the values of a, b, and c.

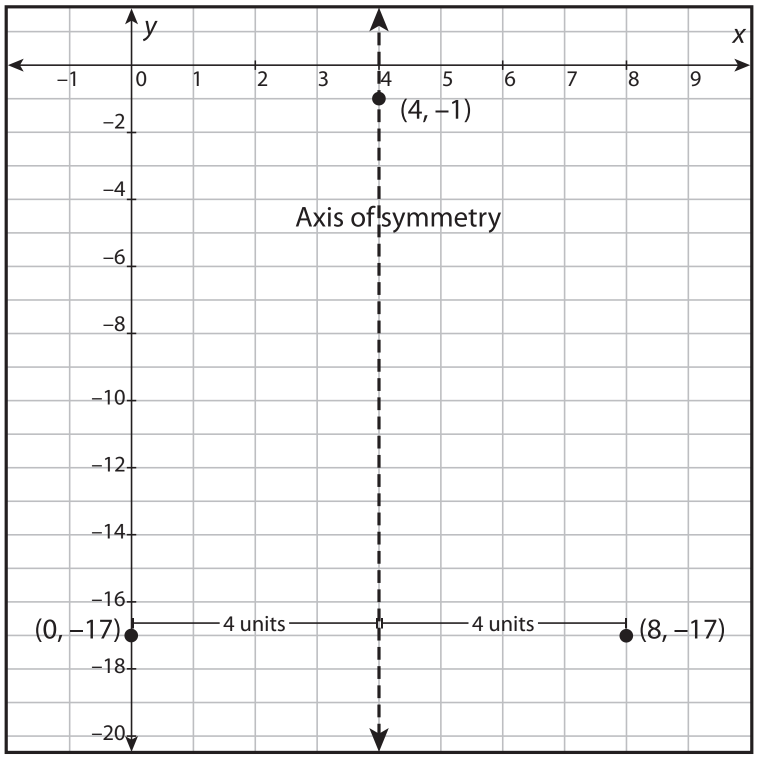
   $a = -1$, $b = 8$, and $c = -17$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$	Quadratic formula
$x=\dfrac{-(8)\pm\sqrt{(8)^2-4(-1)(-17)}}{2(-1)}$	Substitute –1 for a, 8 for b, and –17 for c.
$x=\dfrac{-8\pm\sqrt{-4}}{-2}$	Simplify.

   In this case, the discriminant, –4, is negative, which means there are no real solutions.

   This also means that there are no x-intercepts.

5. Plot the points from steps 2–4 and their symmetric points over the axis of symmetry.



For a more accurate graph, determine an additional pair of symmetric points.

Choose any x-value on the left or right of the axis of symmetry.

Evaluate the function for the chosen value of x to determine the output value.

Let's choose $x = 1$.

$g(x) = -x^2 + 8x - 17$	Original equation
$g(1) = -(1)^2 + 8(1) - 17$	Substitute 1 for x.
$g(1) = -10$	Simplify.

(1, –10) is an additional point on the parabola.

Plot (1, –10) on the same graph.

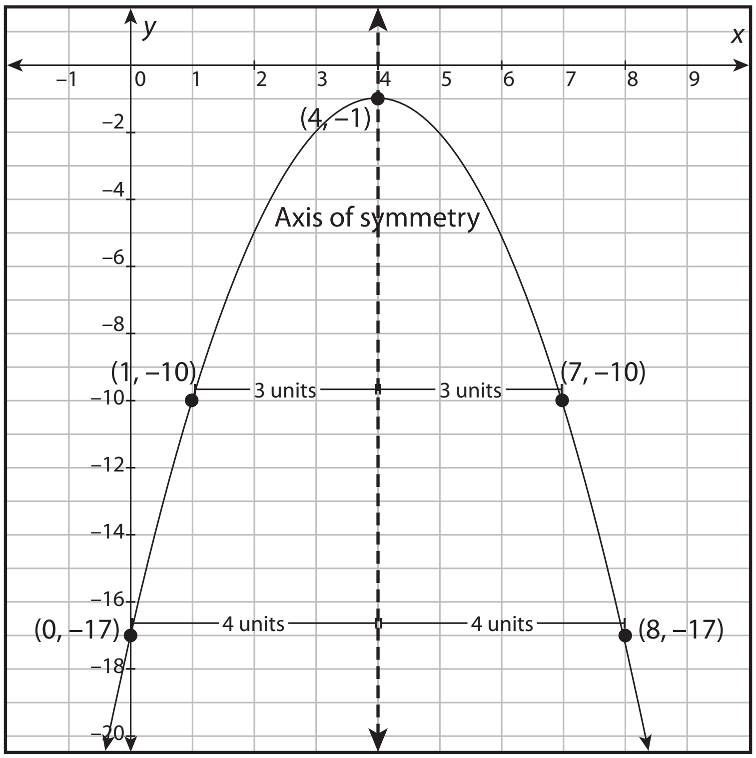
(1, –10) is 3 units from the axis of symmetry.

Locate the point that is symmetric to the point (1, –10) with respect to the axis of symmetry.

(continued)

(7, –10) is also 3 units from the axis of symmetry and is symmetrical to the original point (1, –10) with respect to the axis of symmetry.

You can verify that (1, –10) and (7, –10) are the same distance from the axis of symmetry and are also symmetrical by referring to the graph.



Example 5

Create the equation of a quadratic function given a vertex of (2, –4) and a *y*-intercept of 4.

1. Write an equation for b in terms of a.

   Set the x-coordinate of the vertex equal to $\frac{-b}{2a}$.

$2=\frac{-b}{2a}$	Substitute 2 for x.
$4a = -b$	Multiply both sides by $2a$.
$-4a = b$	Multiply both sides by –1.
$b = -4a$	

2. Substitute the expression for b from step 1, the coordinates of the vertex, and the y-intercept into the standard form of a quadratic equation.

$y = ax^2 + bx + c$	Standard form of a quadratic equation
$y = ax^2 + (-4a)x + c$	Substitute $-4a$ for b.
$(-4) = a(2)^2 + (-4a)(2) + c$	Substitute the vertex (2, –4) for x and y.
$(-4) = a(2)^2 + (-4a)(2) + 4$	Substitute the y-intercept of 4 for c.
$-4 = 4a - 8a + 4$	Simplify, then solve for a.
$-8 = -4a$	
$a = 2$	

3. Substitute the value of a into the equation for b from step 1.

$b = -4a$	Equation from step 1
$b = -4(2)$	Substitute 2 for a.
$b = -8$	Simplify.

4. Substitute a, b, and c into the standard form of a quadratic equation.

$y = ax^2 + bx + c$	Standard form of a quadratic equation
$y = 2x^2 - 8x + 4$	Substitute 2 for a, –8 for b, and 4 for c.

The equation of the quadratic function with a vertex of (2, –4) and a y-intercept of 4 is $y = 2x^2 - 8x + 4$.