Guided Practice 5.8

Example 1

Determine the equation of a quadratic function in standard form, given the zeros $x = 2$ and $x = -2$, and the point (0, 3) that lies on the graph of the function.

1. Write the zeros as expressions equal to 0 to determine the factors of the quadratic equation.

   Recall that when solving a quadratic equation in factored form, you take all factors and set them equal to 0 to determine the solution.

   To work backward, we must undo the solving process to find the factors of the equation.

$x = 2$	$x = -2$
$x - 2 = 0$	$x + 2 = 0$

   The factors of the equation are $(x - 2)$ and $(x + 2)$, so the equation is $f(x) = a(x - 2)(x + 2)$.

2. Use the point (0, 3) to find the value of a.

   The equation $f(x) = a(x - 2)(x + 2)$ is written in intercept form, or $f(x) = a(x - p)(x - q)$.

   Substitute the coordinates of the point (0, 3) into the equation to find a.

$f(x) = a(x - 2)(x + 2)$	Equation
$3 = a(0 - 2)(0 + 2)$	Substitute the point (0, 3) for x and $f(x)$.
$3 = a(-2)(2)$	Simplify, then solve for a.
$3 = -4a$	
$-\frac{3}{4} = a$	

   The value of a is $-\frac{3}{4}$.

3. Write the equation in standard form.

   Insert the value found for a into the equation from step 1 and solve.

$f(x) = a(x - 2)(x + 2)$	Equation
$f(x) = -\frac{3}{4}(x-2)(x+2)$	Substitute $-\frac{3}{4}$ for a.
$f(x) = -\frac{3}{4}(x^2 - 4)$	Simplify by multiplying the binomial factors.
$f(x) = -\frac{3}{4}x^2 + 3$	Distribute.

   The equation of the quadratic function in standard form, with zeros $x = 2$ and $x = -2$, and the point (0, 3), is $f(x) = -\frac{3}{4}x^2 + 3$.

Example 2

Identify the x-intercepts, if any, the equation of the axis of symmetry, and the vertex of the quadratic function $f(x) = (x + 5)(x + 2)$. Use this information to graph the function.

1. Identify the x-intercepts.

   The x-intercepts of the quadratic function are the zeros of the function.

   Start by setting $f(x)$ equal to 0 and solving to determine the zeros.

$f(x) = (x + 5)(x + 2)$	Original equation
$0 = (x + 5)(x + 2)$	Set $f(x)$ equal to 0.
$0 = x + 5$ or $0 = x + 2$	Set each factor equal to 0.
$-5 = x$ or $-2 = x$	Solve each equation for x.

2. Determine the equation of the axis of symmetry to find the vertex.

   The axis of symmetry is the line that divides the parabola in half.

   To determine its equation, find the midpoint of the two zeros and draw the vertical line through that point.

   Insert the zeros into the formula $x=\frac{p+q}{2}$ to find the midpoint.

   Let $p = -5$ and $q = -2$.

   $$x=\frac{-5+(-2)}{2}=\frac{-7}{2}=-3.5$$

   The equation of the axis of symmetry is $x = -3.5$.

   The axis of symmetry extends through the vertex of the parabola. This means that the vertex (h, k) of the parabola has an x-coordinate of –3.5.

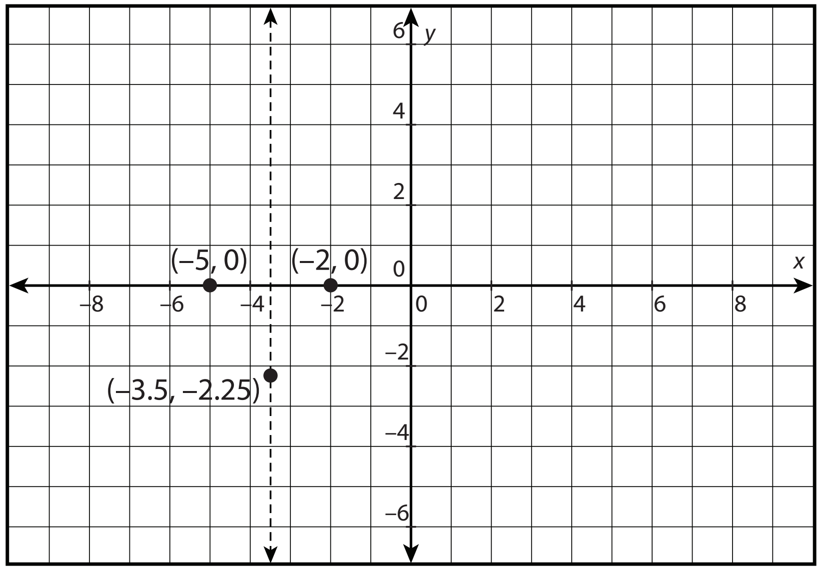
   Substitute –3.5 for x in the original equation to determine the y-coordinate of the vertex.

$f(x) = (x + 5)(x + 2)$	Original equation
$f(-3.5) = (-3.5 + 5)(-3.5 + 2)$	Substitute –3.5 for x.
$f(-3.5) = (1.5)(-1.5)$	Simplify.
$f(-3.5) = -2.25$	

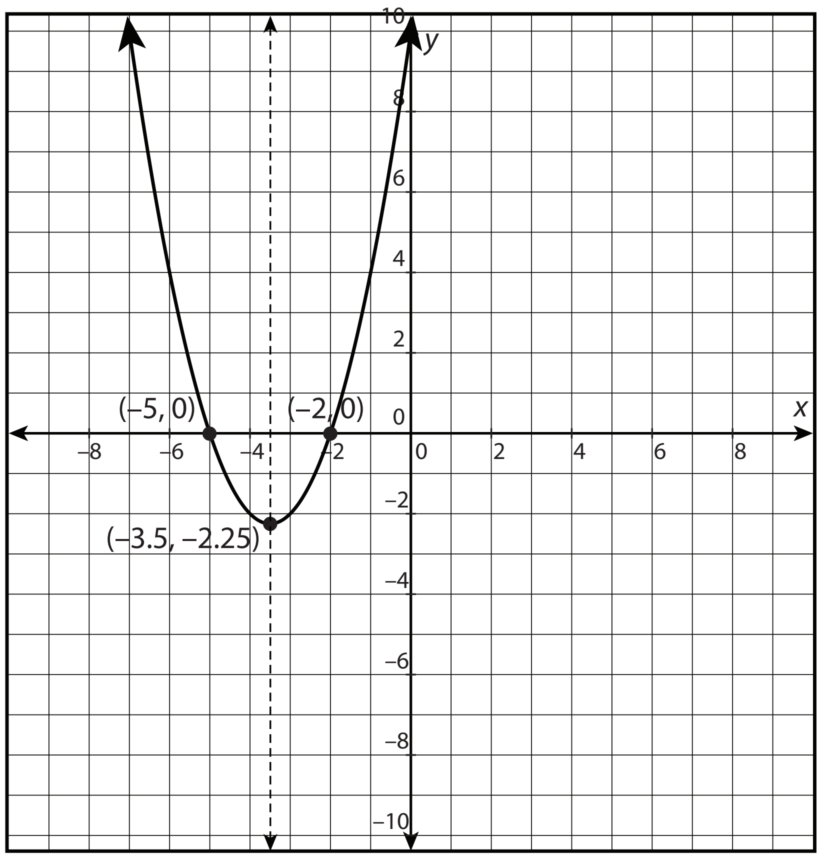
   The y-coordinate of the vertex is –2.25.

   The vertex is the point (–3.5, –2.25).

3. Graph the zeros, the vertex, and the axis of symmetry of the given equation.

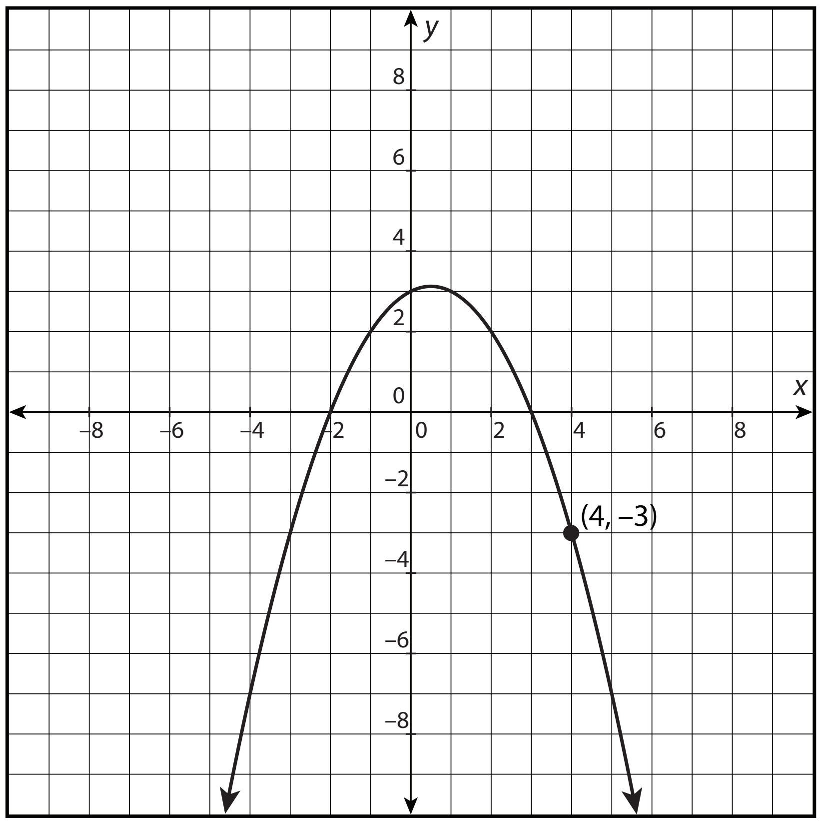


4. Sketch the function based on the zeros and the vertex.



Example 3

Use the *x*-intercepts and the graphed point to write the equation of the function in standard form.



1. Find the *x*-intercepts of the graph and write them as zeros of the function.

   Identify the *x*-intercepts and write them as zeros to prepare to write the factors of the equation.

   The *x*-intercepts are –2 and 3.

   Therefore, the zeros are $x = -2$ and $x = 3$.

2. Write the zeros as expressions set equal to 0 to determine the factors of the quadratic equation.

$x = -2$	$x = 3$
$x + 2 = 0$	$x - 3 = 0$

The factors of the quadratic equation are $(x + 2)$ and $(x - 3)$, so the equation is $f(x) = a(x + 2)(x - 3)$.

3. Use the point (4, –3) to find the value of a.

The equation $f(x) = a(x + 2)(x - 3)$ is written in intercept form, or $f(x) = a(x - p)(x - q)$.

Substitute (4, –3) into the equation to find a.

$f(x) = a(x + 2)(x - 3)$	Equation
$-3 = a(4 + 2)(4 - 3)$	Substitute (4, –3) for x and $f(x)$.
$-3 = a(6)(1)$	Simplify.
$-3 = 6a$	
$-\frac{1}{2} = a$	

The value of a is $-\frac{1}{2}$.

4. Write the equation in standard form.

Insert the value found for a into the equation from step 2 and solve.

$f(x) = a(x + 2)(x - 3)$	Equation
$f(x) = -\frac{1}{2}(x+2)(x-3)$	Substitute $-\frac{1}{2}$ for a.
$f(x) = -\frac{1}{2}(x^2 - x - 6)$	Multiply the binomials.
$f(x) = -\frac{1}{2}x^2 + \frac{1}{2}x + 3$	Distribute $-\frac{1}{2}$ over the parentheses.

The standard form of the equation is $f(x) = -\frac{1}{2}x^2 + \frac{1}{2}x + 3$.