Guided Practice 5.9

Example 1

Suppose that the flight of a launched bottle rocket can be modeled by the function $f(x) = -(x - 1)(x - 6)$, where $f(x)$ measures the height above the ground in meters and x represents the horizontal distance in meters from the launching spot at the point (1, 0). How far does the bottle rocket travel in the horizontal direction from launch to landing? What is the maximum height the bottle rocket reaches? How far has the bottle rocket traveled horizontally when it reaches its maximum height? Graph the function.

1. Identify the x-intercepts of the function.

   In the function, $f(x)$ represents the height of the bottle rocket. At launch and landing, the height of the bottle rocket is 0.

   The function $f(x) = -(x - 1)(x - 6)$ is of the form $f(x) = a(x - p)(x - q)$, where p and q are the x-intercepts.

   The x-intercepts of the function are 1 and 6.

   Find the distance between the two points to determine how far the bottle rocket traveled in the horizontal direction.

   $6 - 1 = 5$

   The bottle rocket traveled 5 meters in the horizontal direction from launch to landing.

2. Determine the maximum height of the bottle rocket.

   The maximum height occurs at the vertex.

   Find the x-coordinate of the vertex using the formula $x = \dfrac{p+q}{2}$.

$x = \dfrac{p+q}{2}$	Formula to determine the x-coordinate of the vertex of a parabola
$x = \dfrac{6+1}{2}$	Substitute 6 for p and 1 for q.
$x = 3.5$	Simplify.

   The x-coordinate of the vertex is 3.5.

(continued)

Use this value to determine the vertex of the function.

$f(x) = -(x - 1)(x - 6)$	Original function
$f(3.5) = -[(3.5) - 1][(3.5) - 6]$	Substitute 3.5 for x.
$f(3.5) = -(2.5)(-2.5)$	Simplify.
$f(3.5) = 6.25$	Multiply.

The y-coordinate of the vertex is 6.25.

The maximum height reached by the bottle rocket is 6.25 meters.

3. Determine the horizontal distance from the launch point to the maximum height of the bottle rocket.

   We know that the bottle rocket is launched from the point (1, 0) and reaches a maximum height at (3.5, 6.25). Subtract the x-value of the two points to find the distance traveled horizontally.

   $3.5 - 1 = 2.5$

   Another method is to take the total distance traveled horizontally from launch to landing and divide by 2 to find the same answer. This is because the maximum value occurs halfway between the x-intercepts of the function.

   $\frac{5}{2} = 2.5$

   The bottle rocket travels 2.5 meters horizontally when it reaches its maximum.

4. Graph the function.

Use a graphing calculator or complete a table of values. Use the x-intercepts and vertex as three of the known points. Choose x-values on either side of the vertex for two additional x-values.

x	y
1	0
2	
3.5	6.25
5	
6	0

To determine the y-coordinates of the additional points, substitute each x-value into the original function and evaluate.

$f(x) = -(x - 1)(x - 6)$	Original function
$f(2) = -[(2) - 1][(2) - 6]$	Substitute 2 for x.
$f(2) = -(1)(-4)$	Simplify.
$f(2) = 4$	Multiply.

$f(x) = -(x - 1)(x - 6)$	Original function
$f(5) = -[(5) - 1][(5) - 6]$	Substitute 5 for x.
$f(5) = -(4)(-1)$	Simplify.
$f(5) = 4$	Multiply.

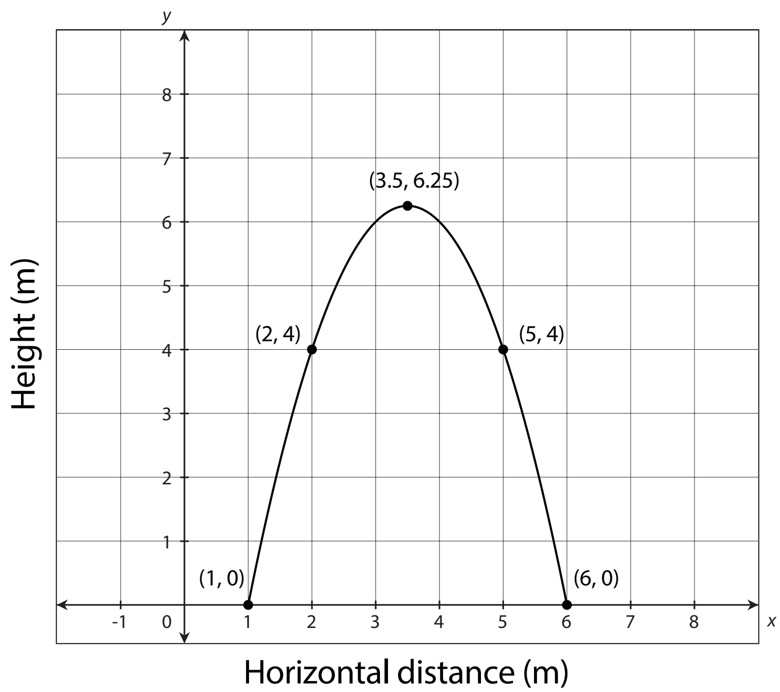
Fill in the missing table values.

x	y
1	0
2	**4**
3.5	6.25
5	**4**
6	0

(continued)

Plot the points on a coordinate plane and connect them using a smooth curve.

Since the function models the flight of a bottle rocket, it is important to only show the portion of the graph where both horizontal distance and height are positive.



Example 2

Reducing the cost of an item can result in a greater number of sales. The revenue function that predicts the revenue in dollars, $R(x)$, for each \$1 change in price, x, for a particular item is $R(x) = -100(x - 7)^2 + 28{,}900$. What is the maximum value of the function? What does the maximum value mean in the context of the problem? What price increase maximizes the revenue and what does it mean in the context of the problem? Graph the function.

1. Determine the maximum value of the function.

   The function $R(x) = -100(x - 7)^2 + 28{,}900$ is written in vertex form, $f(x) = a(x - h)^2 + k$, where (h, k) is the vertex.

   The vertex of the function is (7, 28,900); therefore, the maximum value is 28,900.

2. Determine what the maximum value means in the context of the problem.

   The maximum value of 28,900 means that the maximum revenue resulting from increasing the price by x dollars is \$28,900.

3. Determine the price increase that will maximize the revenue and what it means in the context of the problem.

   The maximum value occurs at the vertex, (7, 28,900).

   This means an increase in price of \$7 will result in the maximum revenue.

4. Graph the function.

   Use a graphing calculator or complete a table of coordinates. Use the vertex as one known point. Choose *x*-values on either side of the vertex to have four additional *x*-values.

x	y
0	
5	
7	28,900
9	
14	

   To determine the *y*-coordinates of the additional points, substitute each *x*-value into the original function and solve.

$R(x) = -100(x - 7)^2 + 28{,}900$	Original function
$R(0) = -100[(0) - 7]^2 + 28{,}900$	Substitute 0 for *x*.
$R(0) = 24{,}000$	Simplify.

$R(x) = -100(x - 7)^2 + 28{,}900$	Original function
$R(5) = -100[(5) - 7]^2 + 28{,}900$	Substitute 5 for *x*.
$R(5) = 28{,}500$	Simplify.

$R(x) = -100(x - 7)^2 + 28{,}900$	Original function
$R(9) = -100[(9) - 7]^2 + 28{,}900$	Substitute 9 for *x*.
$R(9) = 28{,}500$	Simplify.

$R(x) = -100(x - 7)^2 + 28{,}900$	Original function
$R(14) = -100[(14) - 7]^2 + 28{,}900$	Substitute 14 for *x*.
$R(14) = 24{,}000$	Simplify.

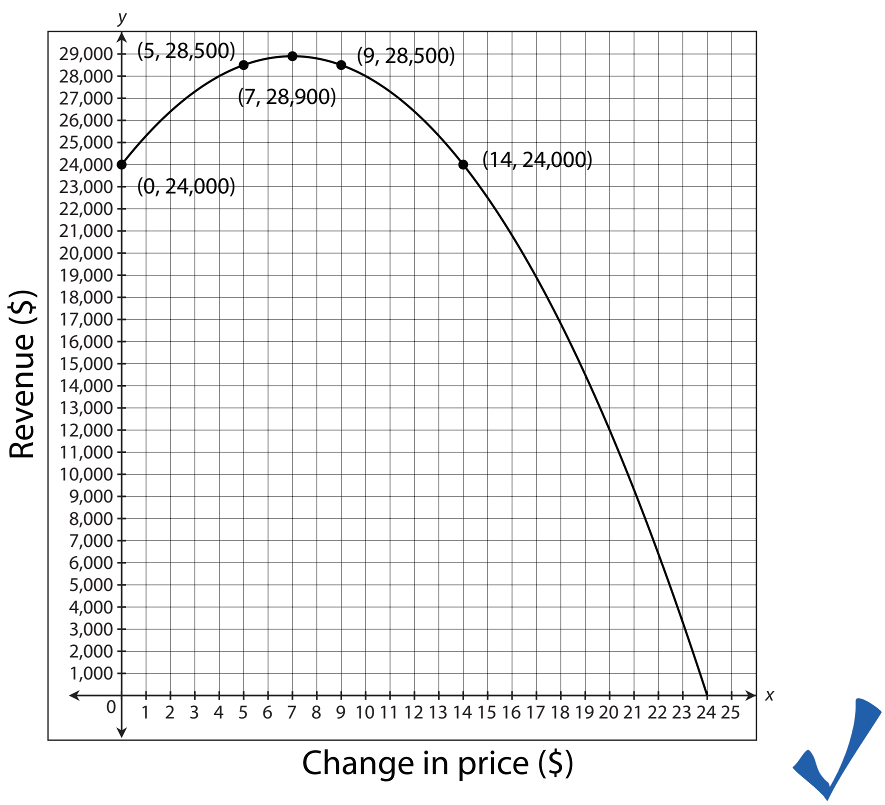
(*continued*)

Fill in the missing table values.

x	y
0	**24,000**
5	**28,500**
7	28,900
9	**28,500**
14	**24,000**

Notice that the points (0, 24,000) and (14, 24,000) are the same horizontal distance from the vertex on either side. The same is true for (5, 28,500) and (9, 28,500).

Plot the points on a coordinate plane and connect using a smooth curve.



Example 3

A football is kicked and follows a path given by $f(x) = -0.03x^2 + 1.8x$, where $f(x)$ represents the height of the ball in feet and x represents the horizontal distance in feet. What is the maximum height the ball reaches? What horizontal distance maximizes the height? Graph the function.

1. Determine the maximum height of the ball.

   The function $f(x) = -0.03x^2 + 1.8x$ is written in standard form, $f(x) = ax^2 + bx + c$, where $a = -0.03$, $b = 1.8$, and $c = 0$.

   The maximum occurs at the vertex, $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

   Determine the x-coordinate of the vertex.

$x = \frac{-b}{2a}$	Formula to determine the x-coordinate of the vertex of a parabola
$x = \frac{-(1.8)}{2(-0.03)}$	Substitute values for a and b.
$x = 30$	Simplify.

   Determine the y-coordinate of the vertex.

$f(x) = -0.03x^2 + 1.8x$	Original function
$f(30) = -0.03(30)^2 + 1.8(30)$	Substitute 30 for x.
$f(30) = 27$	Simplify.

   The vertex is (30, 27), so the maximum value is 27 feet.

   The maximum height the ball reaches is 27 feet.

2. Determine the horizontal distance of the ball when it reaches its maximum height.

   This horizontal distance is determined by the x-coordinate of the vertex.

   The vertex is (30, 27).

   The ball will have traveled 30 feet in the horizontal direction when it reaches its maximum height.

3. Graph the function.

Use a graphing calculator or complete a table of coordinates. Use the vertex as one known point. Choose x-values on either side of the vertex to have four additional x-values.

x	y
5	
20	
30	27
40	
55	

To determine the y-coordinates of the additional points, substitute each x-value into the original function and solve.

$f(x) = -0.03x^2 + 1.8x$	Original function
$f(5) = -0.03(5)^2 + 1.8(5)$	Substitute 5 for x.
$f(5) = 8.25$	Simplify.

$f(x) = -0.03x^2 + 1.8x$	Original function
$f(20) = -0.03(20)^2 + 1.8(20)$	Substitute 20 for x.
$f(20) = 24$	Simplify.

$f(x) = -0.03x^2 + 1.8x$	Original function
$f(40) = -0.03(40)^2 + 1.8(40)$	Substitute 40 for x.
$f(40) = 24$	Simplify.

$f(x) = -0.03x^2 + 1.8x$	Original function
$f(55) = -0.03(55)^2 + 1.8(55)$	Substitute 55 for x.
$f(55) = 8.25$	Simplify.

(*continued*)

Fill in the missing table values.

x	*y*
5	**8.25**
20	**24**
30	27
40	**24**
55	**8.25**

Notice that the points (5, 8.25) and (55, 8.25) are the same horizontal distance from the vertex on either side. The same is true for (20, 24) and (40, 24).

Plot the points on a coordinate plane and connect them using a smooth curve.

Since the function models the path of a kicked football, it is important to only show the portion of the graph where both height and horizontal distance are positive.