UNIT 2 LESSON 3

INTERPRETING THE AVERAGE RATE OF CHANGE

The average rate of change for a function can be calculated by determining

$$m = \frac{\Delta y}{\Delta x}\text{, or the } \frac{\text{change in y}}{\text{change in x}}$$

Specific input-output boundaries will be used to find the average rate of change. Using function notation, for the interval $a < x < b$, the average rate of change is calculated by the

formula $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

***a & b represent the input or the given interval

EX #1) To find a, b, $f(a)$, and $f(b)$ from a table, use the points on the boundary of the requested average rate of change. For example, to find the average rate of change between years 2 and 5 in the following table, use the points (2, 80) and (5, 160), so $f(b) = 160$, $f(a) = 80$, $b = 5$, and $a = 2$:

Years	0	1	2	3	4	5	6	7
Value	50	70	80	120	110	160	170	250

The interval given is $2 < x < 5$ or [2, 5] for the years. The interval is the input/domain of the function (x-coordinate).

Find the y-coordinate of the function in the table (value):

At year 2, the value is 80 → so this is the coordinate (2, 80)

At year 5, the value is 160 → so this is the coordinate (5, 160)

NOW FIND THE AVERAGE RATE OF CHANGE (slope) $\frac{y_2 - y_1}{x_2 - x_1} = \frac{160-80}{5-2} = 26.67$

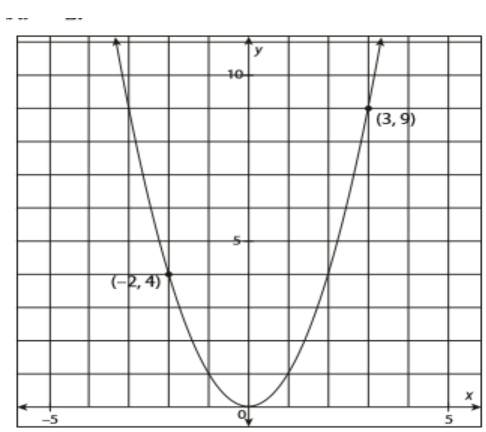
Ex #2) To find the values of a, b, $f(a)$, and $f(b)$ from a graph, use the coordinate points on the boundary of the requested average rate of change. For example, to find the average rate of change for $-2 < x < 3$ in the following graph, use the points (–2, 4) and (3, 9), so $f(b) = 9$, $f(a) = 4$, $b = 3$, and $a = -2$:

The interval given is $-2 < x < 3$ or [-2, 3]. The interval is the input/domain of the function (x-coordinate).

Find the y-coordinate of the function in the graph (where does the graph meet the x-coordinate given):

The coordinates based on the graph (-2, 4) and (3, 9)

NOW FIND THE AVERAGE RATE OF CHANGE (slope) $\frac{y_2 - y_1}{x_2 - x_1} = \frac{9-4}{3-(-2)} = \frac{5}{5} = 1$



EX #3) To find the values of a, b, $f(a)$, and $f(b)$ from a function rule or equation, evaluate the function at the input values of the boundary of the requested average rate of change to find the corresponding output values. For example, to find the average rate of change for $0 < x < 8$ for the function $f(x) = 2(1.5)^x$:

- $f(0) = 2(1.5)^0 = 2$ Evaluate the function for $f(0)$.
- $f(0) = 2(1.5)^8 = 51.258$ Evaluate the function for $f(8)$.
- Use the pairs (0, 2) and (8, 51.258), so $f(b) = 51.258$, $f(a) = 2$, $b = 8$, and $a = 0$.

In this example the interval has been given: 0 < x < 8 or [0, 8]

Find the y-coordinate by substituting each interval into the function and solve.

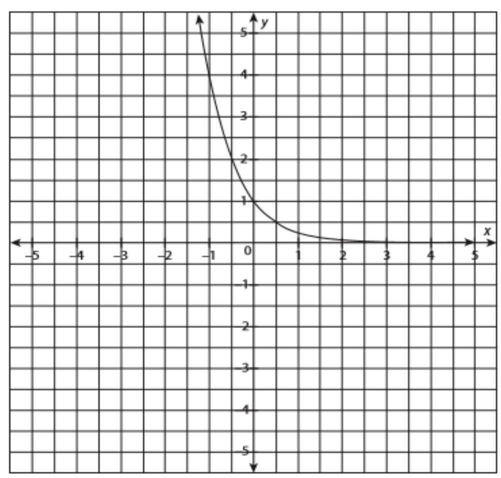
The coordinates will be (0, 2) and (8, 51.258) ****work is shown above

NOW FIND THE AVERAGE RATE OF CHANGE (slope) $\frac{y_2-y_1}{x_2-x_1} = \frac{51.258-2}{8-0} = 6.157$

EX #4) Find the average rate of change over the interval (–1, 0). What does the average rate of change tell you about the function on the interval? Does the rate of change for the function appear to increase, decrease, or remain the same as x increases greater than 0?

The interval given is -1 < x < 0 or [-1, 0]. The interval is the input/domain of the function (x-coordinate).

Find the y-coordinate of the function in the graph (where does the graph meet the x-coordinate given):



The coordinates based on the graph (-1, 4) and (0, 1)

NOW FIND THE AVERAGE RATE OF CHANGE (slope):

$$\frac{y_2-y_1}{x_2-x_1} = \frac{1-4}{0-(-1)} = \frac{-3}{1} = -3$$

EX #5) YOU TRY!! Use the table below to determine the rate of change for the interval [15, 20].

Weeks (x)	Amount owed in dollars ($f(x)$)
0	1700
5	1575
10	1450
15	1325
20	1200